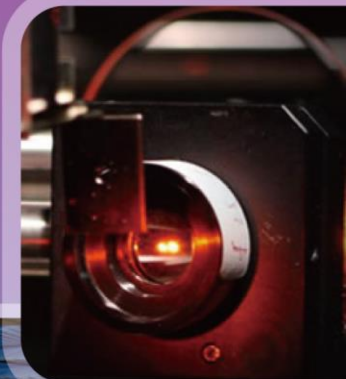
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electronic & Computer Engineering

電 子 及 計 算 機 工 程 學 系



Sequence Design for Acoustic Reflectivity Measurements

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Zhao Licheng
Supervisor: Prof. Daniel Palomar

Collaborator: Ross Murch, Amartansh Dubey, and Li Yue

Reflectivity

- The key thing that needs to be measured is reflectivity. Useful in inverse scattering algorithm.

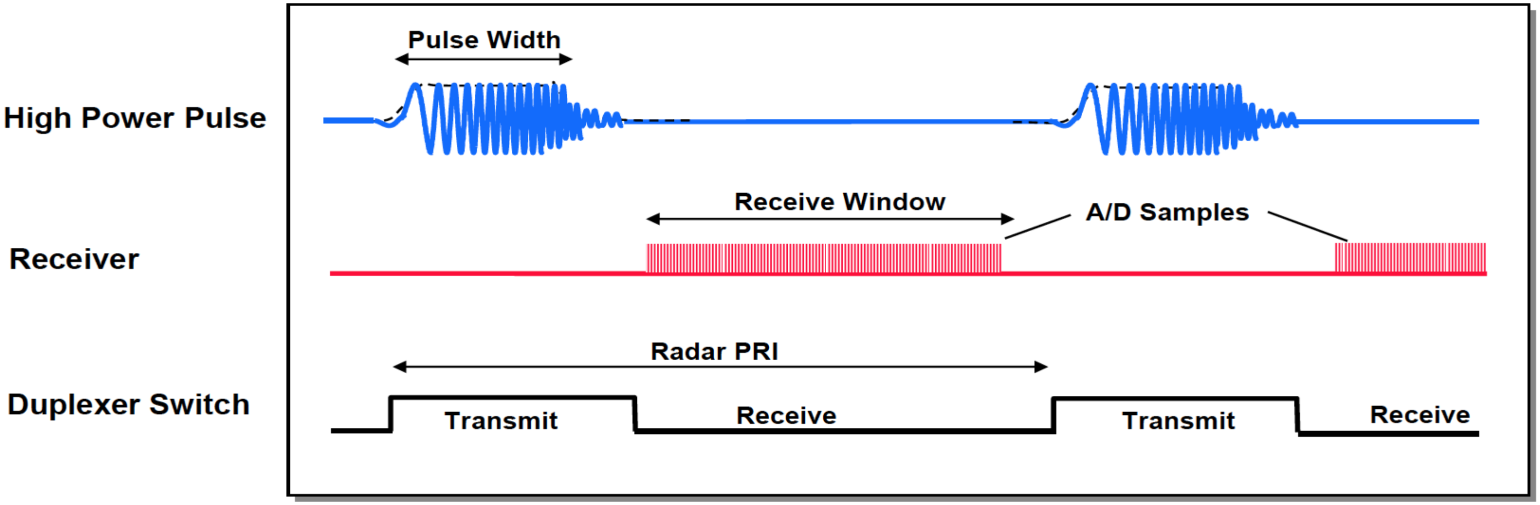
- Suppose we have a single transducer that transmits $V_t(f)$ and the received signal is $V_r(f)$.

- Reflectivity is defined as

$$r(f) = V_r(f)/V_t(f).$$

Sonar Systems

- Measuring reflectivity using a single transducer is equivalent to a sonar system



- **The receiver must be isolated from the powerful transmitter**
  - **Isolation provided by duplexer switching**

Transmitting Sequence Design

- Objective: maximize the quality of reflectivity
- Concerns:
  - Limited transmit power
  - Noise power spectral density, colored noise
  - Frequency response of the transmitter transducer, Required frequency range
  - Required spatial range of the sonar signals
  - Signal separation of the transmitted and received signals so practical sonar systems can be designed

Low Autocorrelation Sequence

- Sequences with low autocorrelations are desired in many digital systems, e.g., coded radar systems and code-division multiple access (CDMA) cellular systems.

- Unimodular (constant modulus) sequences are desired to maximize the transmitted power available in the system.

Low Autocorrelation Sequence

- Let’s define the sequence as

$$\mathbf{x} = [x_1, \cdots, x_N]^T.$$

- The autocorrelation r_k is expressed as

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k} = r_{-k}^*, \ k = 0, \cdots, N-1.$$

- Integrated sidelobe level (ISL)

$$\mathrm{ISL} = \sum_{k=1}^{N-1} |r_k|^2.$$

Low Autocorrelation Sequence

- ISL metric in the frequency domain (Stoica et al., 2009)

$$\mathrm{ISL} = \frac{1}{4N}\sum_{p=1}^{2N}\left[\left|\sum_{n=1}^{N} x_n e^{-j\omega_p(n-1)}\right|^2 - N\right]^2,$$

where

$$\omega_p = \frac{2\pi}{2N}(p-1), p = 1, \cdots, 2N.$$

- ISL minimization problem

$$\begin{array}{ll} \underset{x_n}{\text{minimize}} & \mathrm{ISL} \\ \text{subject to} & |x_n| = 1,\ n = 1, \cdots, N. \end{array}$$

Low Autocorrelation Sequence

- Define

$$\mathbf{x} = [x_1, \cdots, x_N]^T,$$

$$\mathbf{a}_p = \left[1, e^{-j\omega_p}, \cdots, e^{-j\omega_p(N-1)}\right]^T.$$

- The ISL minimization problem is

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \sum_{p=1}^{2N} \left[\mathbf{a}_p^H \mathbf{x}\mathbf{x}^H \mathbf{a}_p - N\right]^2 \\ \text{subject to} \quad & |x_n| = 1,\ n = 1, \cdots, N. \end{aligned}$$

Algorithm

Require: sequence length N

1: Set $k = 0$, initialize $\mathbf{x}^{(0)}$.

2: **repeat**

3: $\mathbf{p}^{(k)} = \left|\mathbf{A}^H \mathbf{x}^{(k)}\right|^2$

4: $p_{\max}^{(k)} = \max_p \{p_p^{(k)} : p = 1, \ldots, 2N\}$

5: $\mathbf{y} = -\mathbf{A}\left(\mathrm{Diag}(\mathbf{p}^{(k)}) - p_{\max}^{(k)}\mathbf{I} - N^2\mathbf{I}\right)\mathbf{A}^H\mathbf{x}^{(k)}$

6: $x_n^{(k+1)} = e^{j\arg(y_n)}, n = 1, \ldots, N$

7: $k \leftarrow k + 1$

8: **until** convergence

[1] J. Song, P. Babu, and D. P. Palomar, "Optimization methods for designing sequences with low autocorrelation sidelobes," IEEE Trans. Signal Process., vol. 63, no. 15, pp. 3998–4009, Aug. 2015.

Spectral Constraint

- In some applications, e.g., cognitive radar, apart from good correlation properties, some spectral constraints are needed to be satisfied.

- For example,

$$\sum_{k\in\Omega} \left|\mathbf{a}_k^H \mathbf{x}\right|^2 \leq \epsilon,$$

i.e., the power in some band should be lower than a threshold.

Spectral Constraint

- **ISL minimization with spectral constraints**

$$\begin{array}{ll} \underset{x_n}{\text{minimize}} & \text{ISL} \\ \text{subject to} & |x_n| = 1,\ n = 1, \cdots, N \\ & \sum_{k \in \Omega} \left| \mathbf{a}_k^H \mathbf{x} \right|^2 \leq \epsilon. \end{array}$$

Spectral Penalty

- For a given $\epsilon > 0$, we can always find a λ such that the problem can be transformed into the following equivalent problem:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \text{ISL} + \lambda \sum_{k \in \Omega} \left| \mathbf{a}_k^H \mathbf{x} \right|^2 \\ \text{subject to} & |x_n| = 1,\ n = 1, \cdots, N. \end{array}$$

- The previous algorithm can be adapted to deal with this problem.

Algorithm

Require: sequence length N, index set Ω and λ.

1: Set $k = 0$, initialize $\mathbf{x}^{(0)}$.

2: **repeat**

3: $$\bar{p}_p^{(k)} = \begin{cases} \left|\mathbf{a}_p^H \mathbf{x}^{(k)}\right|^2 + \lambda/2, & p \in \Omega \\ \left|\mathbf{a}_p^H \mathbf{x}^{(k)}\right|^2, & \text{otherwise} \end{cases}.$$

4: $$\bar{p}_{\max}^{(k)} = \max_p \{\bar{p}_p^{(k)} : p = 1, \ldots, 2N\}$$

5: $$\mathbf{y} = -\mathbf{A}\left(\mathrm{Diag}(\bar{\mathbf{p}}^{(k)}) - \bar{p}_{\max}^{(k)}\mathbf{I} - N^2\mathbf{I}\right)\mathbf{A}^H\mathbf{x}^{(k)}$$

6: $$x_n^{(k+1)} = e^{j\arg(y_n)},\ n = 1, \ldots, N$$

7: $$k \leftarrow k + 1$$

8: **until** convergence

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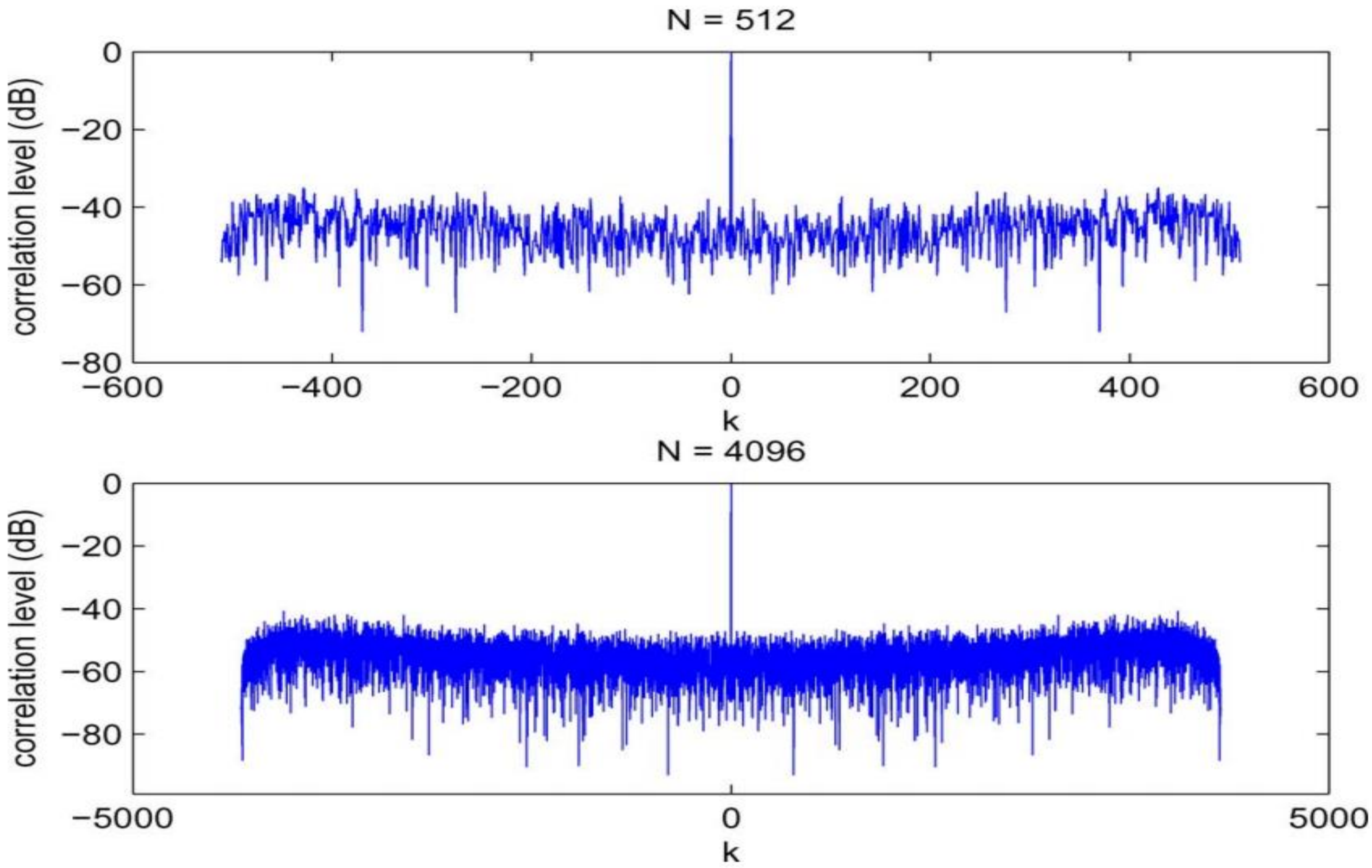
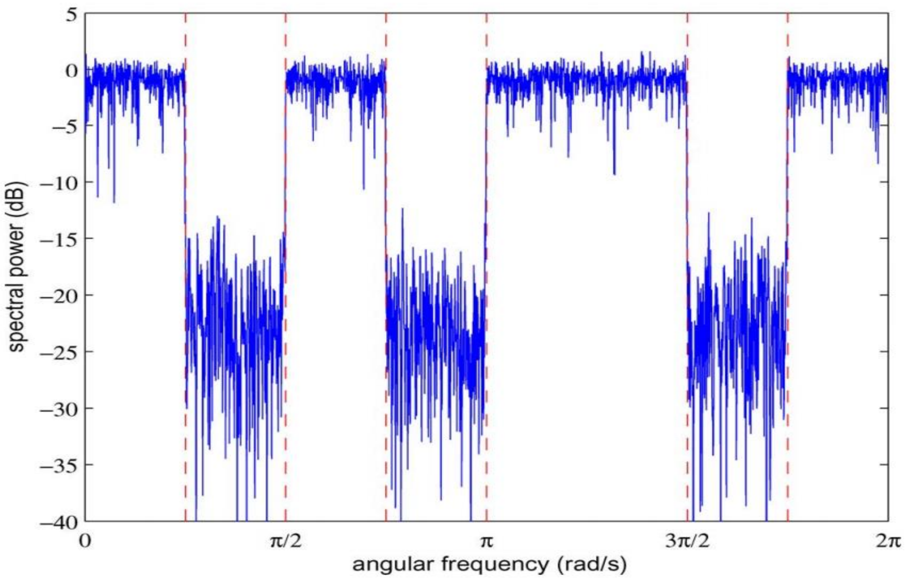
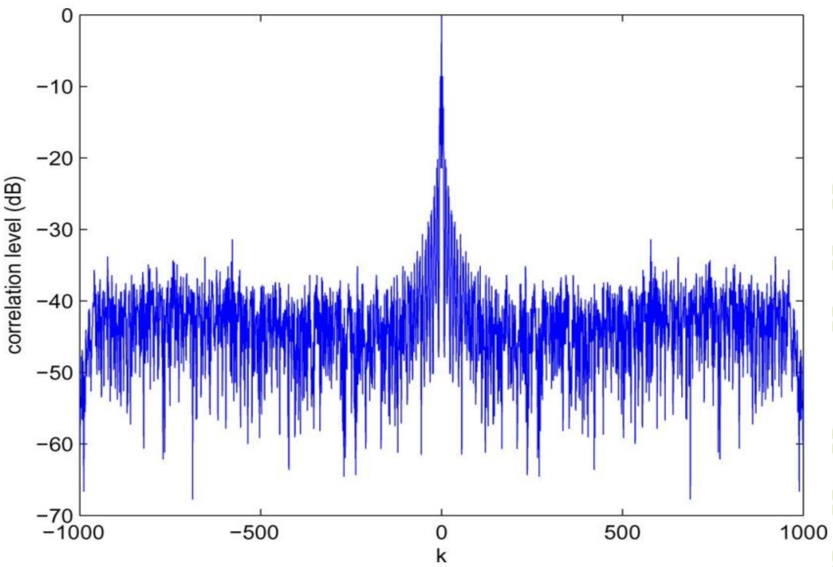
Simulation Result



Simulation Result





Conclusion and Future Plans

- Sequence design is very critical to acoustic reflectivity

- Future plans
  - Maximize SNR (signal to noise ratio) at one location and then do sweeping adaptively
  - Assume a prior distribution on the channel to be estimated and improving the knowledge in successive rounds