Identification of multiple leaks in pipeline: linearized model, maximum likelihood, and super-resolution localization

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Model linearization

Maximum likelihood for detecting multiple leaks

Numerical results

Conclusion
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Model of wave propagation in a pipe

With $N$ leaks at $x^{L_n}$ ($n = 1, \cdots, N$), the head and discharge at a sensor $x^M$:

$$
\begin{pmatrix}
q^M \\
h^M
\end{pmatrix} = M^{NL}(x^M - x^{L_N}) \prod_{n=N}^{1} \left\{ M^L(x^{L_n}) M^{NL}(x^{L_n} - x^{L_{n-1}}) \right\} \begin{pmatrix}
q(x^U) \\
h(x^U)
\end{pmatrix},
$$

Easy to compute, but complex for inverse problem (leak detection) : a $2N$-parameter optimization problem.
Linear approximation of the model

Theorem

Assume that the pipe has $N$ leaks with locations $x^{L_n}$ and sizes $s^{L_n}$, $n = 1, \cdots, N$, then the head and discharge at $x^M$ ($x^M > x^{L_N} > \cdots > x^{L_1}$) is

$$
\begin{pmatrix}
q(x^M) \\
h(x^M)
\end{pmatrix} = \left( M^{NL}(x^M) + \sum_{n=1}^{N} s^{L_n} M^{SL}(L_n, x^{L_n}, x^M) \right) \begin{pmatrix}
q(x^U) \\
h(x^U)
\end{pmatrix} + o \left( \max_{n=1, \cdots, N} (s^{L_n}) \right)
$$

(1)

as $\max_{n=1, \cdots, N} (s^{L_n}) \to 0$. 
Model : numerical justification of the linearized model

\[ C^d A^L / A = 0.0041 \]

\[ C^d A^L / A = 0.0015 \]

Figure – FRF at \( x^M = 1900 \) m. Leak locations : \( x^{L_1} = 400 \) m, \( x^{L_2} = 520 \) m and \( x^{L_3} = 800 \) m. The pipe length is \( l = 2000 \) m.
**Model: numerical justification of linearized model**

![Graph showing mean relative error of FRF w.r.t. leak size](image)

**Figure** – Mean relative error of FRF w.r.t. leak size
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Data: head difference due to leakage ($\Delta h = h^m - h^{NL}$) in the frequency domain by one (or multiple) transducer(s)

\[
\Delta h \approx G(x^L) s^L + n = \sum_{n=1}^{N} G_n(x^{L_n}) s^{L_n} + n. \quad (2)
\]

- $n$: Gaussian independent random noise
- Leak detection (Maximum Likelihood Estimation):

\[
\{\hat{x}^L, \hat{s}^L\} = \arg \min_{x^L, s^L} \| \Delta h - G(x^L)s^L \|^2. \quad (3)
\]
Maximum likelihood for detecting multiple leaks

Estimate the leak locations and leak sizes separately and sequentially:

- The leak locations:

\[ \hat{x}^L = \arg \max_{x^L} \left( \Delta h G^H(x^L) \left( G^H(x^L)G(x^L) \right)^{-1} G^H(x^L) \Delta h \right). \] (4)

- The leak sizes:

\[ s^L = \left( G^H(x^L)G(x^L) \right)^{-1} G^H(x^L) \Delta h. \] (5)
Model linearization

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Numerical results: double leaks

- Two leaks: $x^{L_1} = 300 \ m$, $x^{L_2} = 700 \ m$
- Plot a 2D function (locations of two leaks):

(a) likelihood function

(b) location-size estimate

$\lambda_{\min} = 258 \ m$
Numerical results: double leaks

\[ x^{L_1} = 400 \text{ m}, \quad x^{L_2} = 460 \text{ m} : \quad |x^{L_1} - x^{L_2}| < \lambda_{min}/4 \]

(a) 1-D search (MFP)

(b) ML

Super-resolution localization: two leaks with a distance shorter than \( \lambda_{min}/2 \) can be separately localized

Minimum resolvable distance between leaks: \( 0.5\lambda_{min} \rightarrow 0.15\lambda_{min} \).
Model linearization

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- A linear model of wave propagation in pipe: easy for inverse problem
- Separately estimating the leak location and size
- $N$ leaks: plotting a $N$-D objective function
- Super-resolution: can identify very close leaks
- Any frequency can be used
- Robust with respect to noise and model uncertainties
Thank you very much for your attention!