Identification of multiple leaks in pipeline: linearized model, maximum likelihood, and super-resolution localization

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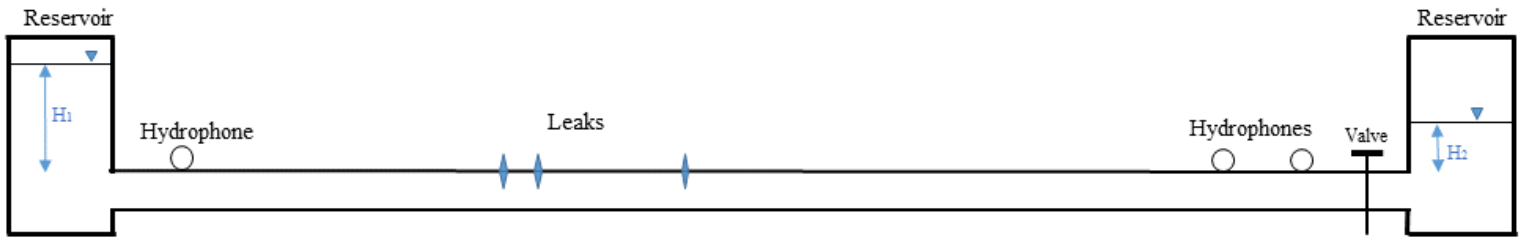
Model of wave propagation in a pipe



With N leaks at x^{L_n} $(n = 1, \cdots, N)$, the head and discharge at a sensor x^M :

$$\begin{pmatrix} q^M \\ h^M \end{pmatrix} = M^{NL}(x^M - x^{L_N}) \prod_{n=N}^{1} \left\{ M^L(x^{L_n}) M^{NL}(x^{L_n} - x^{L_{n-1}}) \right\} \begin{pmatrix} q(x^U) \\ h(x^U) \end{pmatrix},$$

Easy to compute, but complex for inverse problem (leak detection) : a $2N$-parameter optimization problem

Linear approximation of the model

Theorem

Assume that the pipe has N leaks with locations x^{L_n} and sizes s^{L_n}, $n = 1, \cdots, N$, then the head and discharge at x^M $(x^M > x^{L_N} > \cdots > x^{L_1})$ is

$$\begin{pmatrix} q(x^M) \\ h(x^M) \end{pmatrix} = \left(M^{NL}(x^M) + \sum_{n=1}^{N} s^{L_n} M^{SL}(L_n, x^{L_n}, x^M) \right) \begin{pmatrix} q(x^U) \\ h(x^U) \end{pmatrix} + o\left(\max_{n=1,\cdots,N} (s^{L_n}) \right) \tag{1}$$

as $\max_{n=1,\cdots,N} (s^{L_n}) \to 0$.

Model : numerical justification of the linearized model

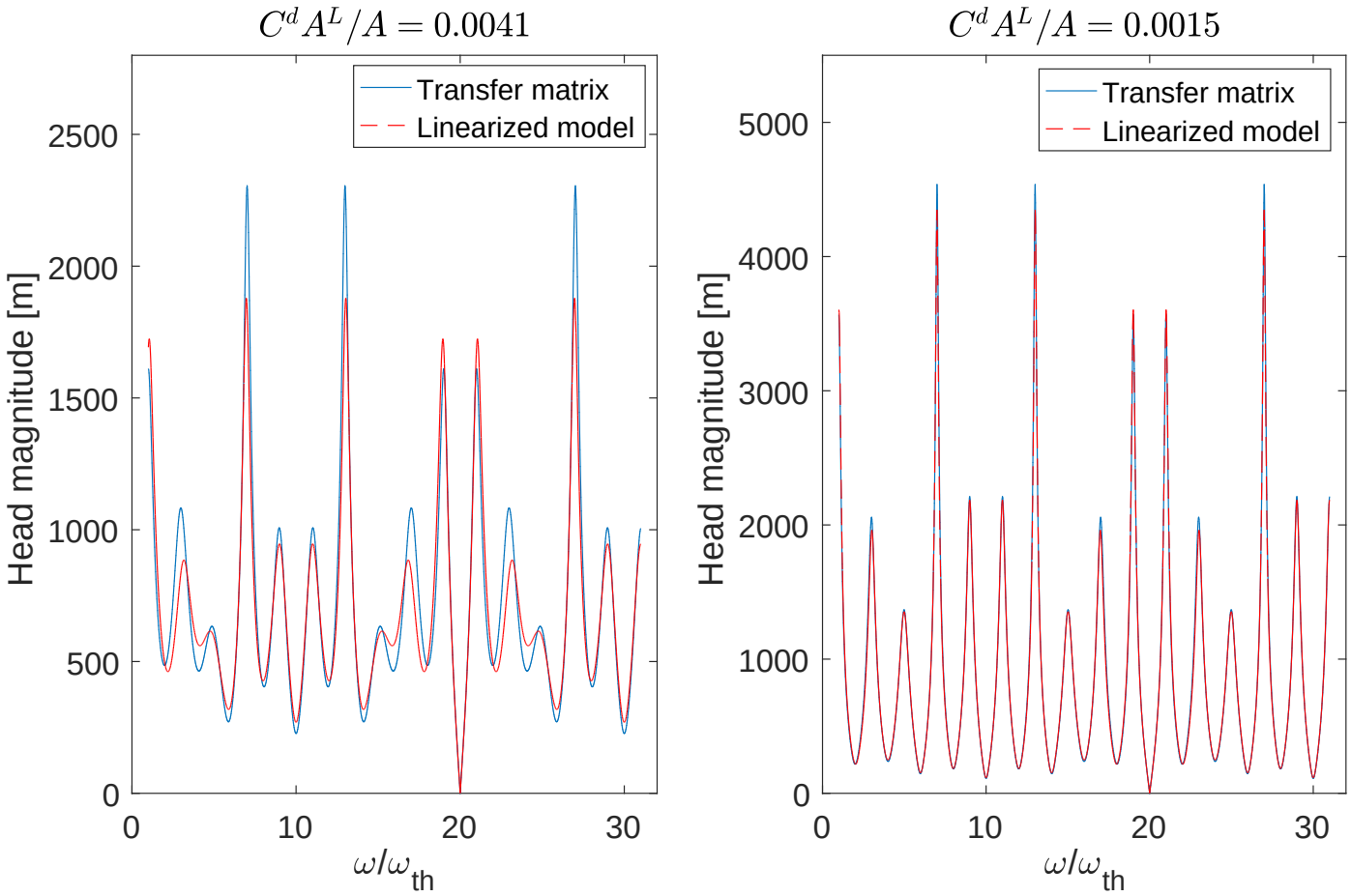


Figure – FRF at $x^M = 1900$ m. Leak locations : $x^{L_1} = 400$ m, $x^{L_2} = 520$ m and $x^{L_3} = 800$ m. The pipe length is $l = 2000$ m.

Model : numerical justification of linearized model

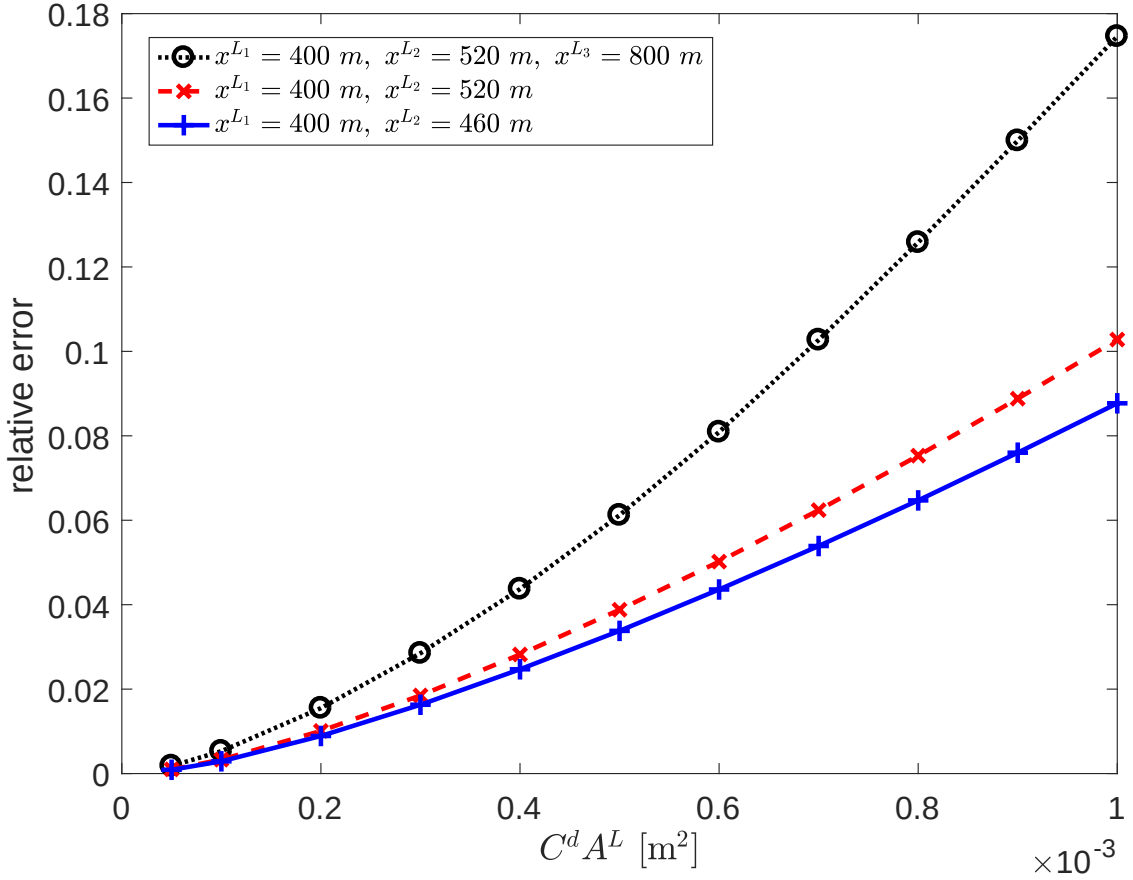


Figure – Mean relative error of FRF w.r.t. leak size

Maximum likelihood for detecting multiple leaks

- Data : head difference due to leakage ($\Delta\mathbf{h} = \mathbf{h}^m - \mathbf{h}^{NL}$) in the frequency domain by one (or multiple) transducer(s)

$$\Delta\mathbf{h} \approx \mathbf{G}(x^L)\mathbf{s}^L + \mathbf{n} = \sum_{n=1}^{N} \mathbf{G}_n(x^{L_n})s^{L_n} + \mathbf{n}. \tag{2}$$

- $\mathbf{n}$: Gaussian independent random noise
- Leak detection (Maximum Likelihood Estimation) :

$$\{\hat{\mathbf{x}}^L, \hat{\mathbf{s}}^L\} = \arg\min_{\mathbf{x}^L, \mathbf{s}^L} \|\Delta\mathbf{h} - \mathbf{G}(\mathbf{x}^L)\mathbf{s}^L\|^2. \tag{3}$$

Maximum likelihood for detecting multiple leaks

Estimate the leak locations and leak sizes separately and sequentially :

- The leak locations :

$$\hat{\mathbf{x}}^L = \arg\max_{\mathbf{x}^L} \left(\Delta\mathbf{h}\mathbf{G}^H(\mathbf{x}^L) \left(\mathbf{G}^H(\mathbf{x}^L)\mathbf{G}(\mathbf{x}^L) \right)^{-1} \mathbf{G}^H(\mathbf{x}^L)\Delta\mathbf{h} \right). \quad (4)$$

  N-dimensional optimization problem : plot this function when $N = 1$ or $N = 2$.

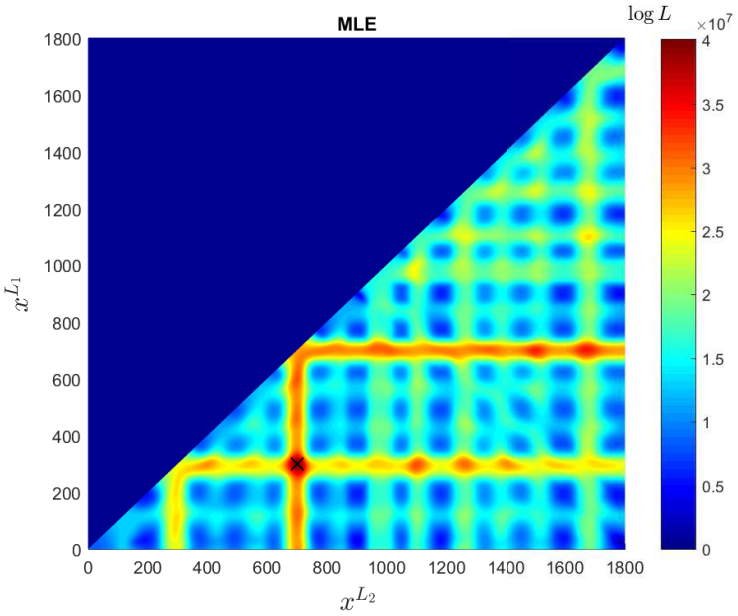
- The leak sizes :

$$\mathbf{s}^L = \left(\mathbf{G}^H(\mathbf{x}^L)\mathbf{G}(\mathbf{x}^L) \right)^{-1} \mathbf{G}^H(\mathbf{x}^L)\Delta\mathbf{h}. \quad (5)$$

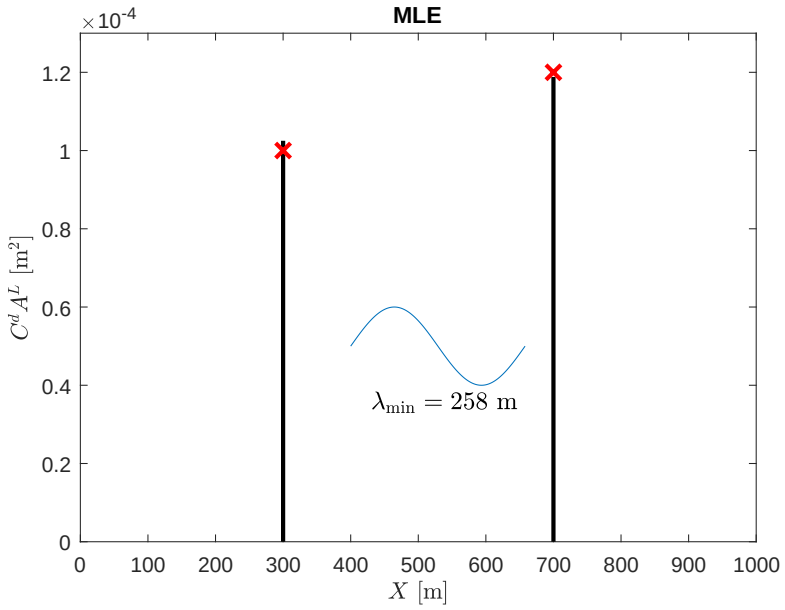
Numerical results : double leaks

- Two leaks : $x^{L_1} = 300\ m$, $x^{L_2} = 700\ m$
- Plot a 2D function (locations of two leaks) :

(a) likelihood function



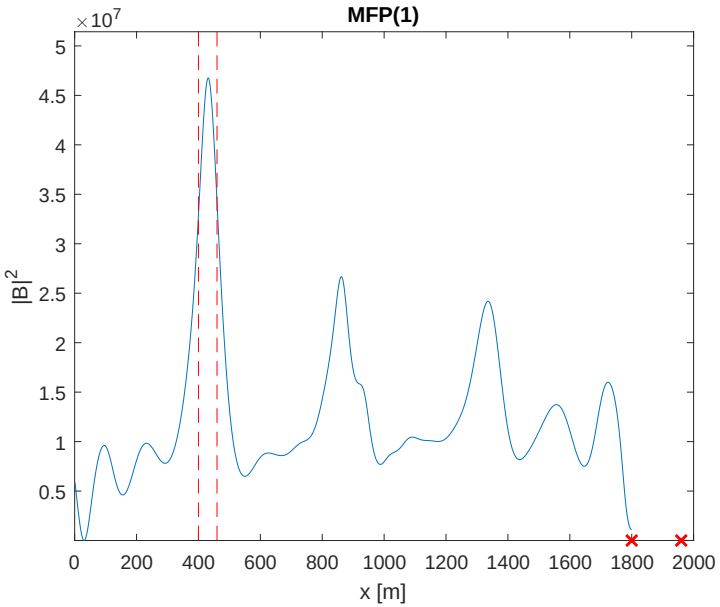
(b) location-size estimate



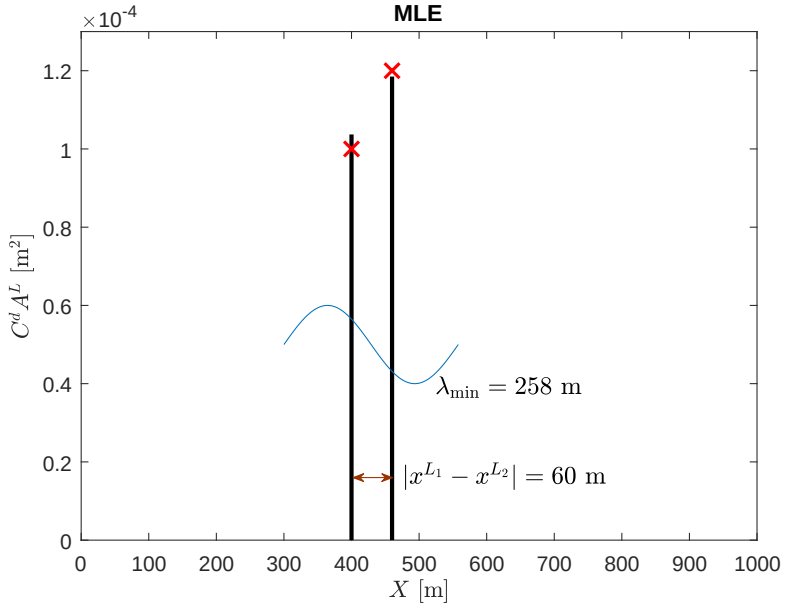
Numerical results : double leaks

$x^{L_1} = 400\ m,\ x^{L_2} = 460\ m : |x^{L_1} - x^{L_2}| < \lambda_{min}/4$

(a) 1-D search (MFP)



(b) ML



Super-resolution localization : two leaks with a distance shorter than $\lambda_{min}/2$ can be separately localized

Minimum resolvable distance between leaks : $0.5\lambda_{min} \rightarrow 0.15\lambda_{min}$.

Conclusion

- A linear model of wave propagation in pipe : easy for inverse problem
- Separately estimating the leak location and size
- N leaks : plotting a N-D objective function
- Super-resolution : can identify very close leaks
- Any frequency can be used
- Robust with respect to noise and model uncertainties

Thank you very much for your attention !