THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electronic & Computer Engineering

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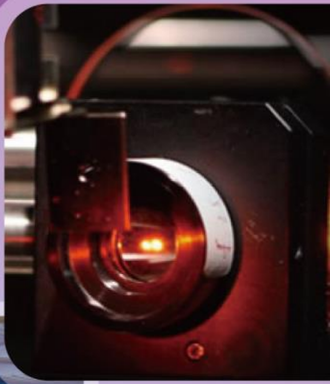

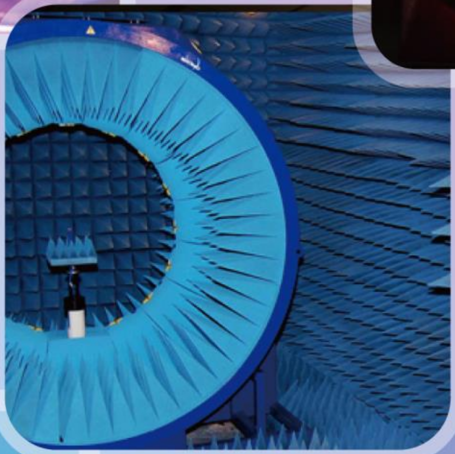

Inverse Scattering: Approximate Methods

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Prof. Murch's group

PhD student since Sept 2016
Inverse scattering

Background

- Exact inverse scattering solutions for 1D wave equations have been around for over 50 years
  - M. Gel' fand and B. M. Levitan, On the determination of a differential equation by its spectral function, Izv. Akad. Nauk SSSR Ser Math, 1951
  - S. Agranovich and V. A. Marchenko, The Inverse Problem of Scattering Theory, 1963
- Further generalization
  - V. E. Zakharov and A. B. Shabat, Sov. Phys. JETP 34, 62 (1972).
- We introduce approximate solution techniques
- We also apply ZS to Water Hammer and Transmission line equations

Background

- The exact methods (Including ZS) typically end up as the solution to Volterra equations and these can be solved using standard numerical methods
- Often difficult to understand the principles involved
- Approximate methods can provide us with
  - Intuition on the working of the process
  - Explicit formula
  - Computational simplicity
  - Are well-posed and more resistant to noise

Wave equations

- Telegrapher's equations

$$\frac{dV(z,t)}{dz} + L(z)\frac{dI(z,t)}{dt} + R(z)I(z,t) = 0$$
$$\frac{dI(z,t)}{dz} + C(z)\frac{dV(z,t)}{dt} + G(z)V(z,t) = 0$$

- Water hammer equations

$$\frac{\partial h^*(x,t)}{\partial x} + \frac{1}{gA(x)}\frac{\partial q^*(x,t)}{\partial t} + Rq^*(x,t) = 0$$
$$\frac{\partial q^*(x,t)}{\partial x} + \frac{gA(x)}{a^2}\frac{\partial h^*(x,t)}{\partial t} = 0$$

- Both sets of equations are essentially the same

Approximations

- Split the total field inside the transmission line into scattered and incident parts
- Rytov approximation
  - Ignore terms which contain spatial derivatives of the scattered field
- Born approximation
  - Approximate the spatial derivative of the total field
- Both lead to expressions for the reconstructed impedance $Z(z) = \sqrt{\frac{L(z)}{C(z)}}$ along the line in terms of the reflection coefficient

Rytov approximation details

- Transform voltage or pressure into logarithmic domain

$$V(x,k) = e^{s(x,k)}$$
$$s(x,k) = s_i(x,k) + s_s(x,k)$$

- Combine with Telegrapher's equations

$$s_s''(x,k) + \left(s_s'(x,k)\right)^2 + 2s_i'(x,k)s_s'(x,k) = \frac{Z'(x)}{Z(x)}\left(s_i'(x,k) + s_s'(x,k)\right)$$

- On introducing Rytov's transformation

$$\tilde{V}_s(x,k) = s_s(x,k)e^{s_i(x,k)}$$

- We obtain

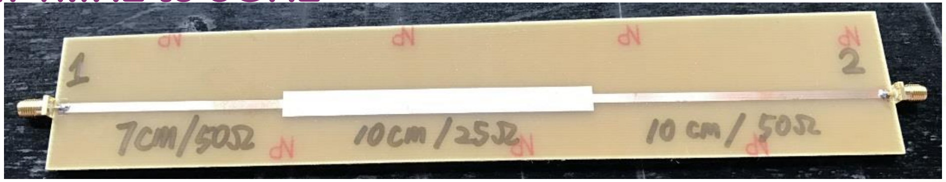
$$\tilde{V}_s''(x,k) + k^2\tilde{V}_s(x,k) = \left[\frac{Z'(x)}{Z(x)}\left(s_i'(x,k) + s_s'(x,k)\right) - \left(s_s'(x,k)\right)^2\right]e^{s_i(x,k)}$$

- Apply Rytov approximation and transform measurement data into Rytov form and solve for Z(x)

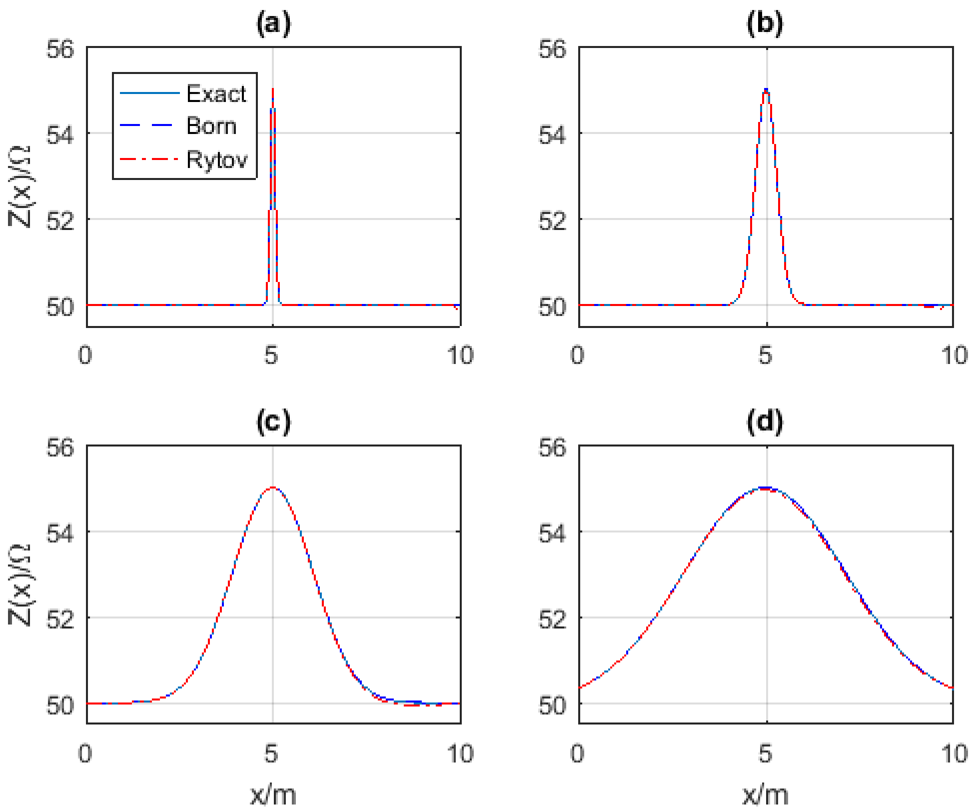
$$\tilde{V}_s(x,k) = V_i(x,k)\ln\left[\frac{V_s(x,k)}{V_i(x,k)} + 1\right]$$

Simulation and Experimental setup

- Simulation setup
  - Lossless
  - Gaussian shaped blockage or impedance profile
  - Transmission line configuration
  - Frequency range 1MHz-8GHz (Step=1MHz)
  - Use middle frequency as reference wavelength of 2.5 cm
  - Profiles from 5 to 160 wavelengths(Phase shift=0.25-8 wavelengths for 5% variation)
  - AWGN added
- Experimental setup
  - Use microstrip transmission lines
  - Collect reflection data from 1MHz to 8GHz
  - Use VNA in wireless lab



Simulation Results- width of impedance



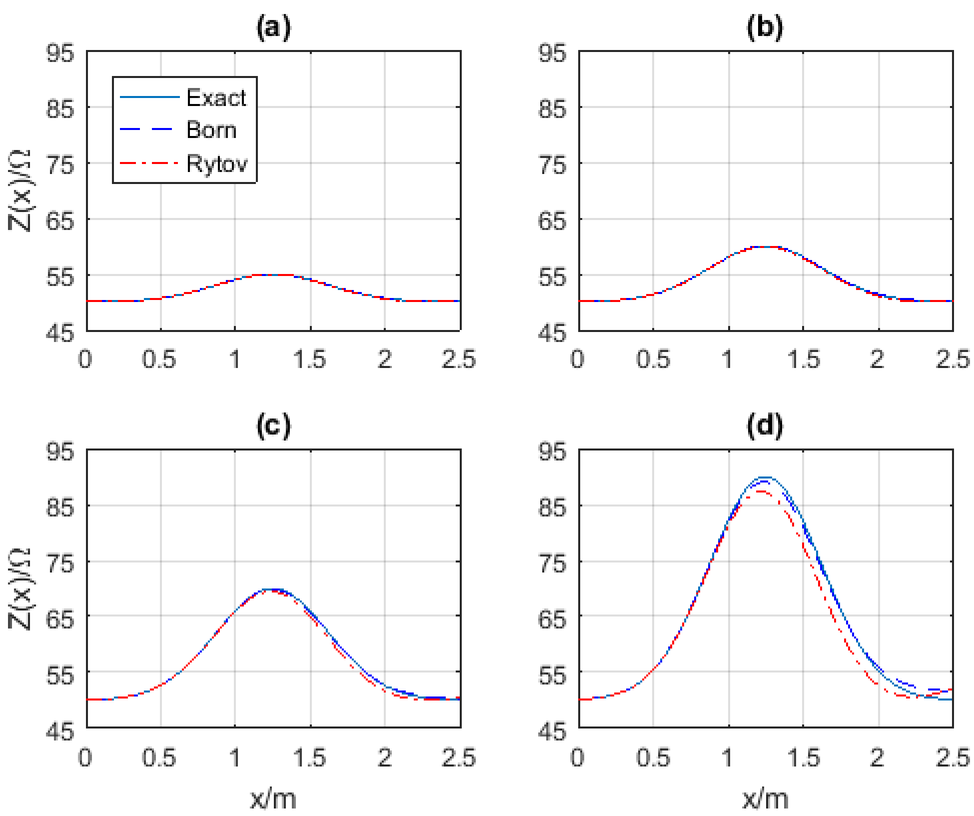
Gaussian-Like Z(x) Profile

$$Z(x) = Z_0 + Z_p e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- $Z_0 = 50\ \Omega$
- $Z_p = 5\ \Omega$

a) $2\sigma = 0.125\ m$
b) $2\sigma = 0.5\ m$
c) $2\sigma = 2\ m$
d) $2\sigma = 4\ m$

Simulation Results- size of impedance



Gaussian-Like Z(x) Profile

$$Z(x) = Z_0 + Z_p e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- $Z_0 = 50\ \Omega$
- $2\sigma = 0.75$ m

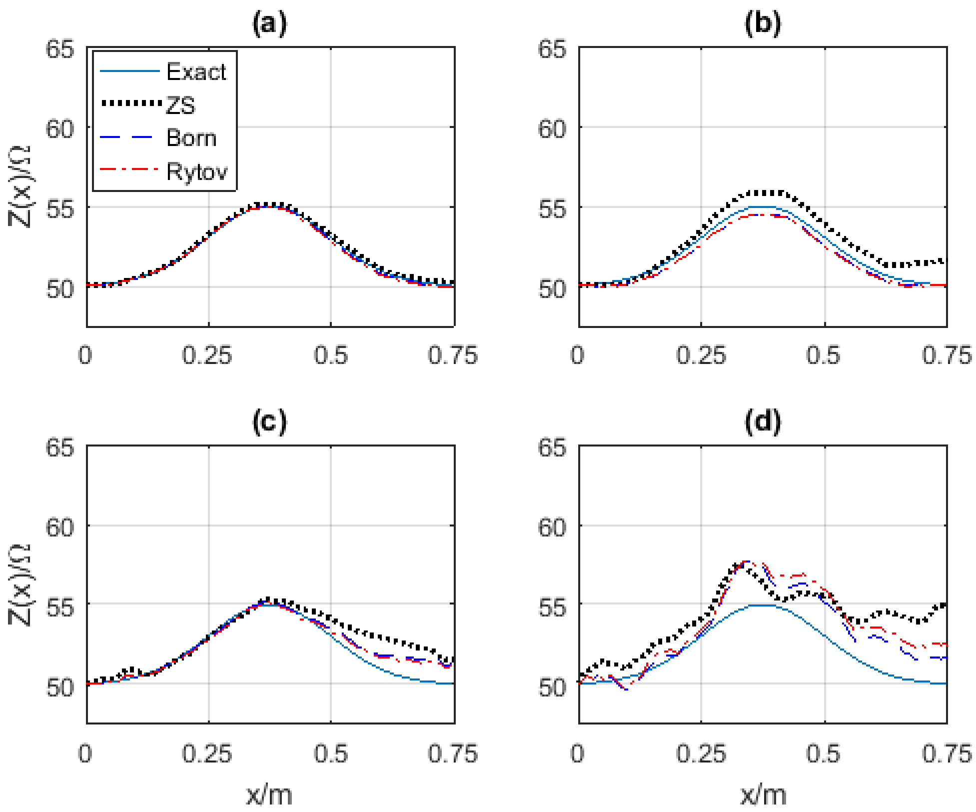
a) $Z_p = 5\ \Omega$

b) $Z_p = 10\ \Omega$

c) $Z_p = 20\ \Omega$

d) $Z_p = 40\ \Omega$

Simulation Results- noise



Gaussian-Like Z(x) Profile

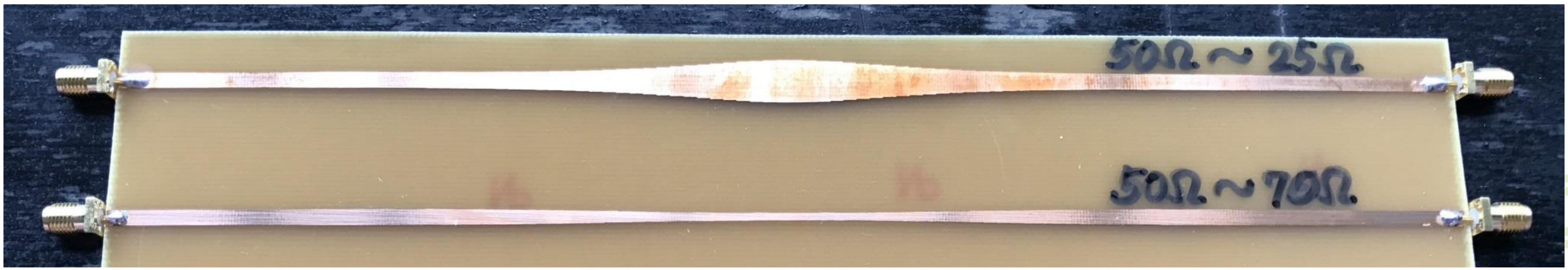
$$Z(x) = Z_0 + Z_p e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- $Z_0 = 50\ \Omega$
- $Z_p = 5\ \Omega$
- $2\sigma = 0.25$ m

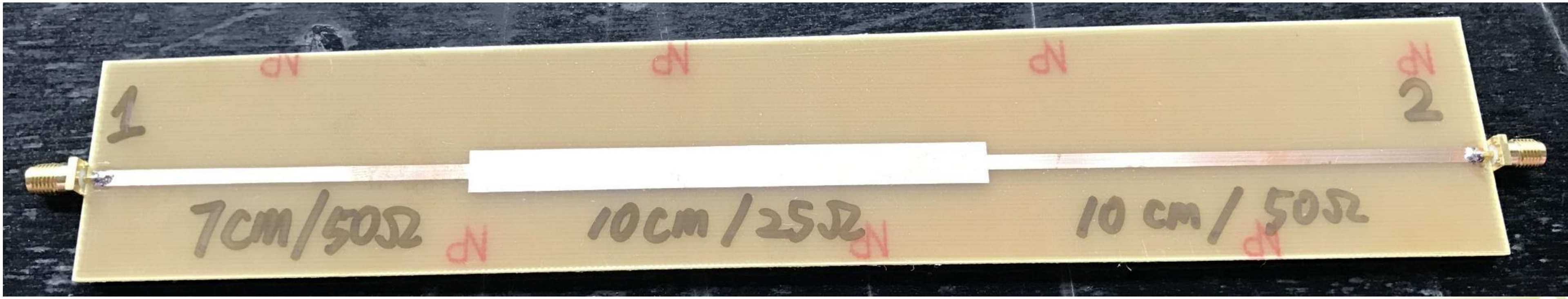
a) SNR=10 dB
b) SNR=3 dB
c) SNR=-3 dB
d) SNR=-10 dB

Experimental Results

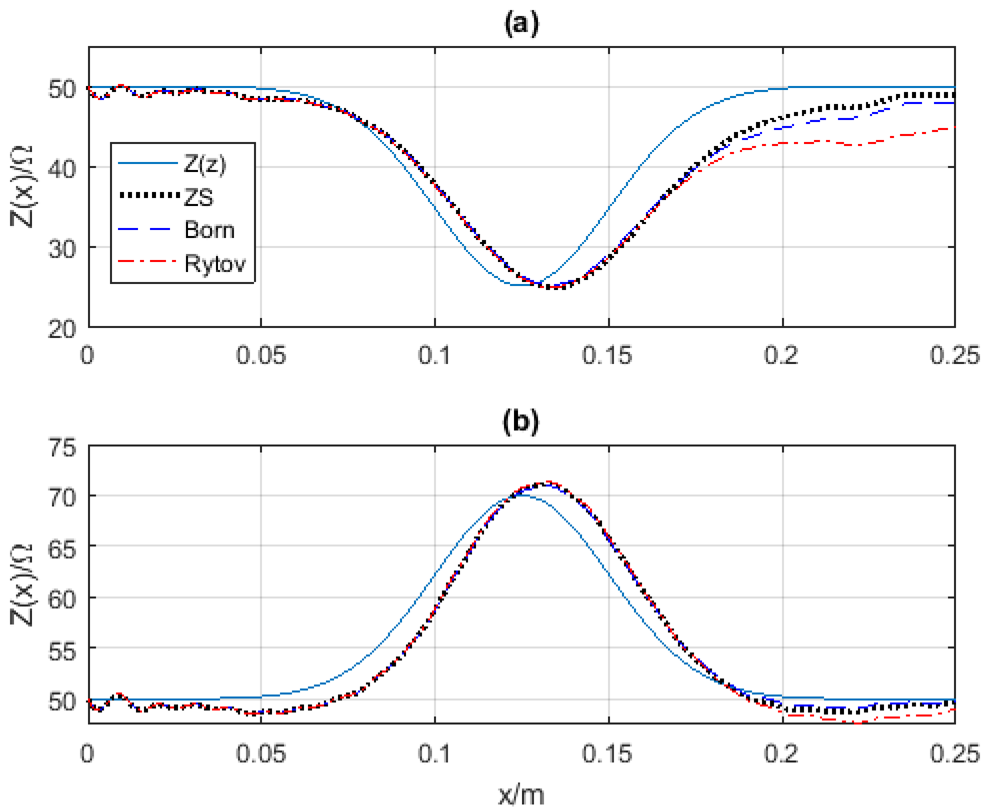
- Gaussian-Like Z(x) Profile(Length=25 cm)
  - $2\sigma = 5\ cm$



- Rectangular-Like Z(x) Profile(Length=27 cm)



Experimental Results- Guassian



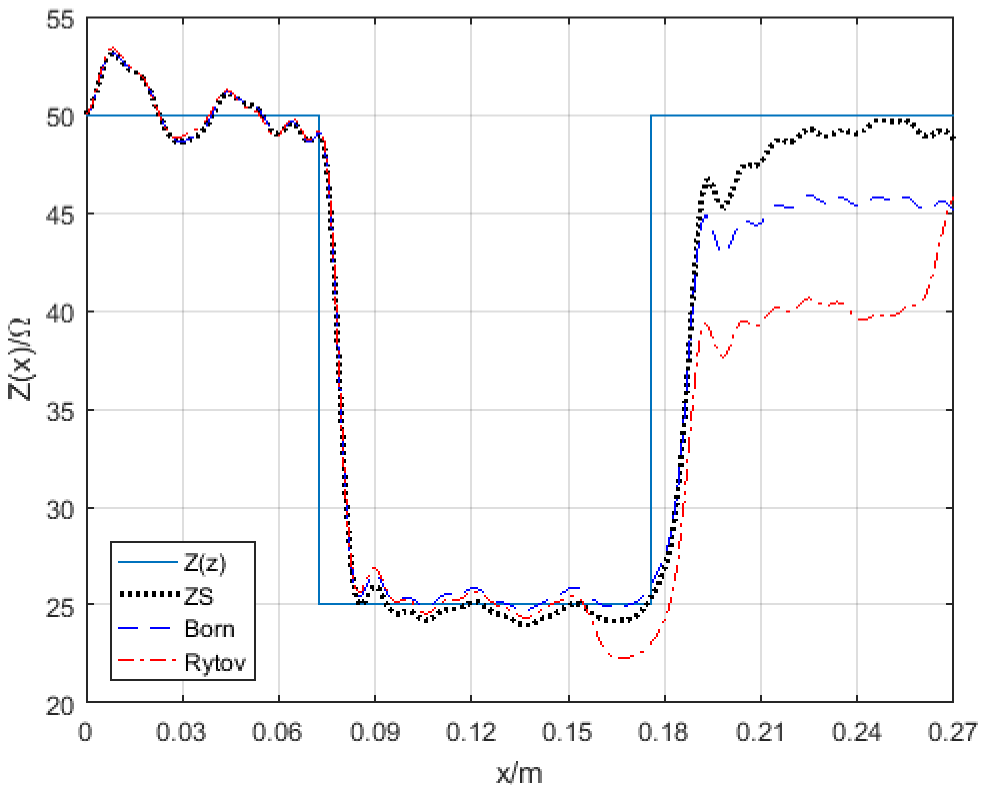
Gaussian-Like Z(x) Profile

$$Z(x) = Z_0 + Z_p e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- $Z_0 = 50\ \Omega$
- $2\sigma = 5\ cm$

a) $Z_p = -25\ \Omega$
b) $Z_p = 20\ \Omega$

Experimental Results- rect



Rectangular-Like Z(x) Profile

1. $7\ cm: 50\ \Omega$
2. $10\ cm: 25\ \Omega$
3. $10\ cm: 50\ \Omega$

Conclusion and Future Plans

- Approximate method provides surprisingly good performance
- Less computational complexity than ZS
- Well-posed- appears more resistant to noise than ZS
- Future plans
  - Explore noise performance in more detail
  - Extend to lossy formulation
  - Gather experimental data from water pipe at both LFW and HFW
  - Compare LFW and HFW performance of exact and approx
  - Work with Dr Pedro Lee on these plans