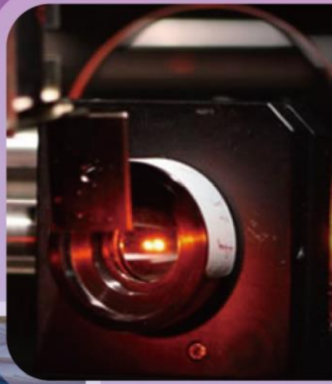
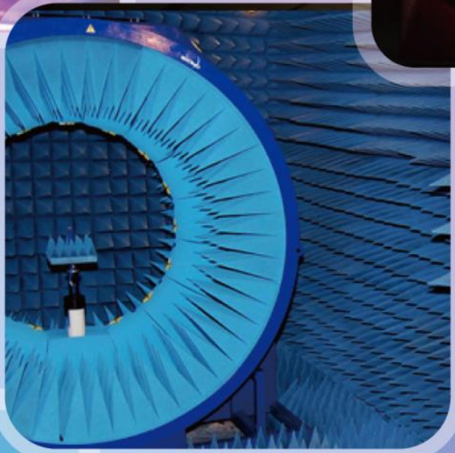
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Department of Electronic & Computer Engineering

電 子 及 計 算 機 工 程 學 系





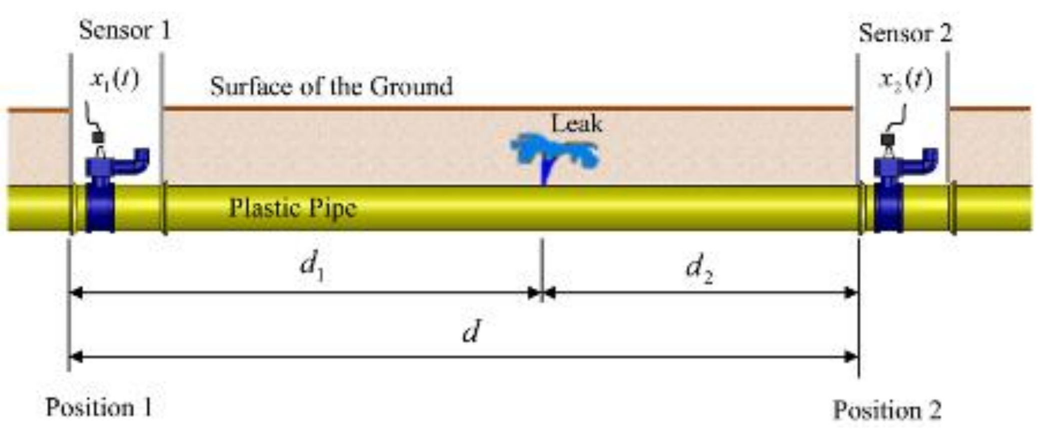
Successive Mode Detection and Parameter Estimation

21 June 2017

Prof. Vincent Lau's group

Background

- We want to detect the pipe leakage or blockage by measuring the frequency domain pressure response at the receiver.



Background

- We have a model which expresses the frequency domain pressure response as a function of system parameters such as the amplitudes and cut off frequencies of different modes.
- These parameters depend on the location/size of the leakage/blockage.
- Therefore, to detect pipe leakage/blockage, we first estimate these parameters from the measured frequency domain pressure response. Then we can infer the location/size of leakage/blockage from the estimated parameters.

Basic Model

- The frequency domain pressure response at the receiver can be expressed as

$$\psi(f, \boldsymbol{\alpha}) = R(f) \sum_{i=1}^{i_{\max}} a_i \phi(f, fcut_i)$$

where

$$\phi(f, fcut_i) = \frac{\exp\left(-1i\frac{2\pi z}{c}\sqrt{(f^2 - fcut_i^2)}\right)}{\frac{2\pi}{c}\sqrt{f^2 - fcut_i^2}} f$$

f is frequency, z,c are recevier to projector distance, wavespeed

$\boldsymbol{\alpha} = [\boldsymbol{a}, \boldsymbol{fcut}]$ is channel parameter vector. $\boldsymbol{a}$ is amplitude vector,$\boldsymbol{fcut}$ is cut off frequency vector.

$R(f)$ is the speaker's frequency response function.

Refined Model with Damping & Reflection Term

$$\psi(f, \boldsymbol{\alpha},) = R(f) \sum_{i=1}^{i_{\max}} \exp\left(-\frac{2\eta_i}{\sqrt{1-(\frac{fcut_i}{f})^2}}\right)\left(a_i\phi_0(f, fcut_i) + a_{i+i_{\max}}\phi_1(f, fcut_i)\right)$$

where

$$\phi_0(f, fcut_i) = \frac{\exp\left(-1i\frac{2\pi z}{c}\sqrt{(f^2 - fcut_i^2)}\right)}{\frac{2\pi}{c}\sqrt{f^2 - fcut_i^2}} f$$

$$\phi_1(f, fcut_i) = \frac{\exp\left(1i\frac{2\pi z}{c}\sqrt{(f^2 - fcut_i^2)}\right)}{\frac{2\pi}{c}\sqrt{f^2 - fcut_i^2}} f$$

$$\boldsymbol{\alpha} = [\boldsymbol{a}, \boldsymbol{fcut}, \boldsymbol{\eta}]. \quad \boldsymbol{a} \in \mathbb{C}^{2i_{\max}}, \quad \boldsymbol{fcut} \in \mathbb{R}_+^{i_{\max}}, \quad \boldsymbol{\eta} \in \mathbb{C}^{i_{\max}}$$

- **The reflection term refers to the propagation change in direction of frequency domain pressure response. It is caused by the signal hitting the blockages and reflecting back.**

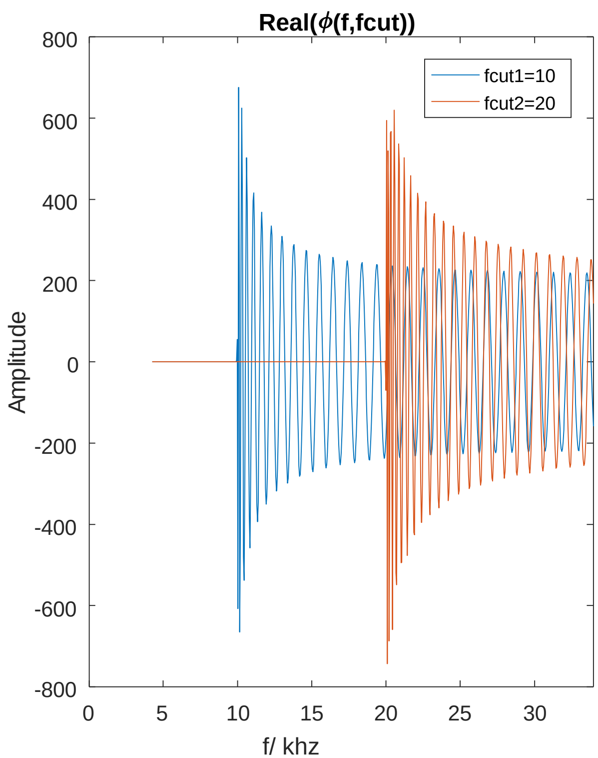
Problem Formulation for Parameter Estimation

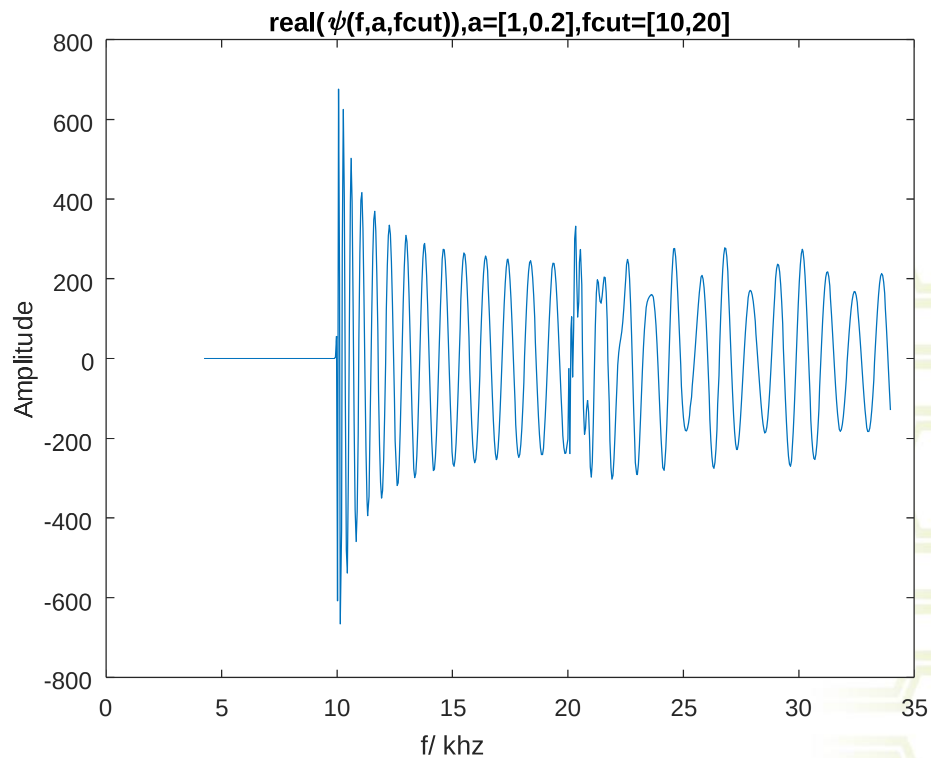
- Measurements $m(f_n)$: Measured frequency domain pressure response in the open air channel
  $f_n = 10\text{hz} : \Delta f : 20\text{khz}$
- The goal is to estimate the channel and speaker parameters from the measurements $m(f_n)$ in the presence of model mismatch and noise
  - note that the 2 different models have different channel parameters
- Objective function:

$$g(\boldsymbol{\alpha}) = \sum_n |\psi(f_n, \boldsymbol{\alpha}) - m(f_n)|^2$$

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} \quad g(\boldsymbol{\alpha})$$

Successive Mode Detection and Parameter Estimation (SMD-PE)





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Basic Idea of the Proposed Algorithm

- Generate the *basis window sequence* for a single mode with cut off frequency τ: a sequence of size Nw, start at τ, step size Δf, generated using the basic model

$$\phi(f_\tau, \tau) = \frac{\exp\left(-1i\frac{2\pi z}{c}\sqrt{(f_\tau^2 - fcut_i^2)}\right.}{\frac{2\pi}{c}\sqrt{f_\tau^2 - \tau^2}} f_\tau \quad , \; f_\tau = \tau : \Delta f : \tau + N\Delta f,$$

- Shift the basis window sequence through the signal and calculate the correlation coefficient sequentially. In order to separate two modes, it is necessary for the window size Nw to be smaller than the gap between two modes

An Illustration of Correlation Calculation for a Signal with Two Modes

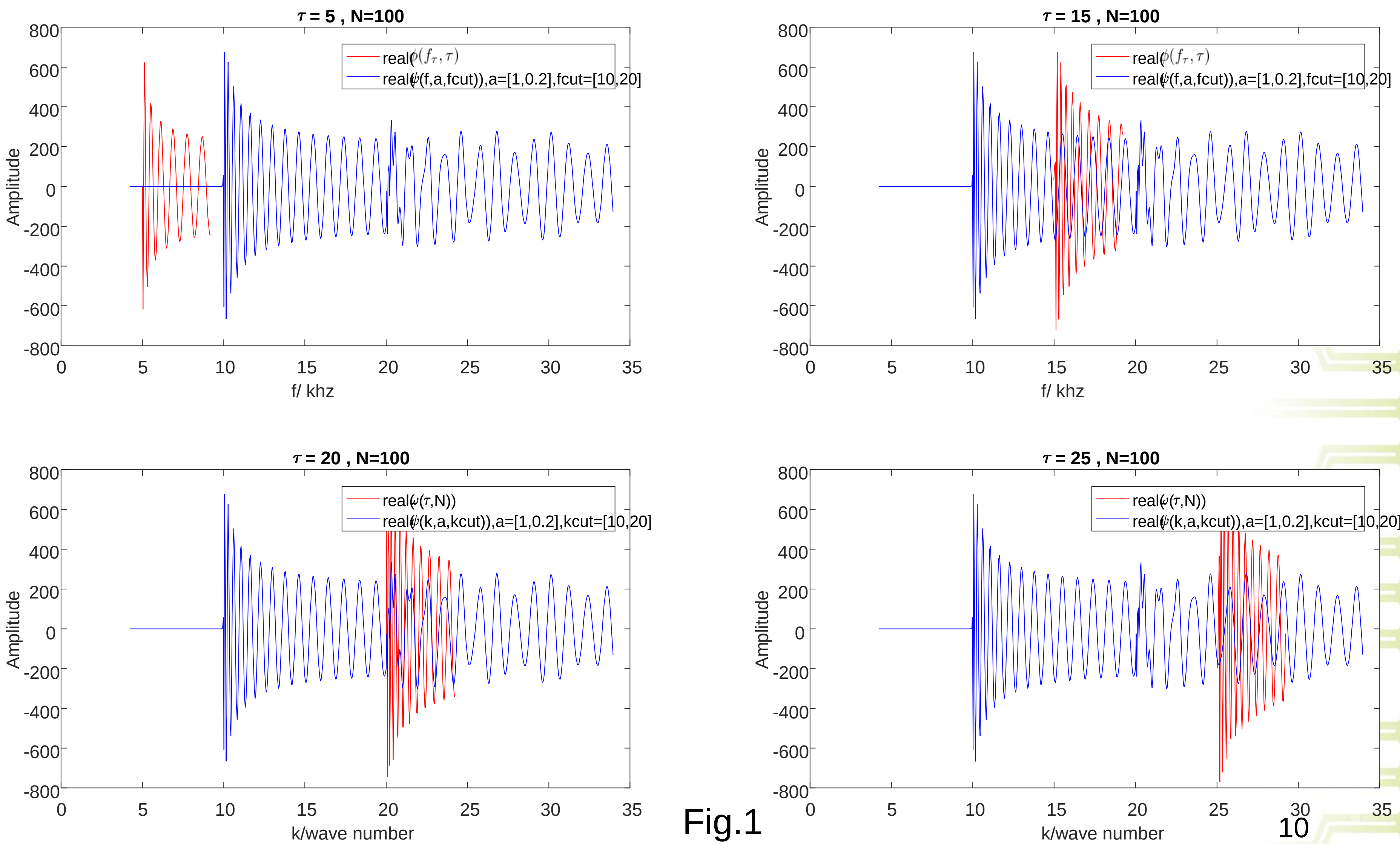


Fig.1

Basic Idea of the Proposed Algorithm

- The correlation coefficient for given τ is:

$$\rho(\tau) = \frac{|<\phi(f_\tau, \tau), m(f_\tau)>|}{||\phi(f_\tau, \tau) m(f_\tau)||}$$

- τ is the translation parameter, determines the location of the window. The window is shifted through the signal $m(f)$

- Fig.2 shows the correlation coefficient by shifting the window through the signal in Fig.1. Two peaks in Fig.1 indicates two dominating modes.

- Two modes can be differentiate from each other since the window size Nw is smaller than the gap between two modes.

- Second mode's peak is weaker than first mode due to interference. Therefore successive interference cancellation is used to increase accuracy.

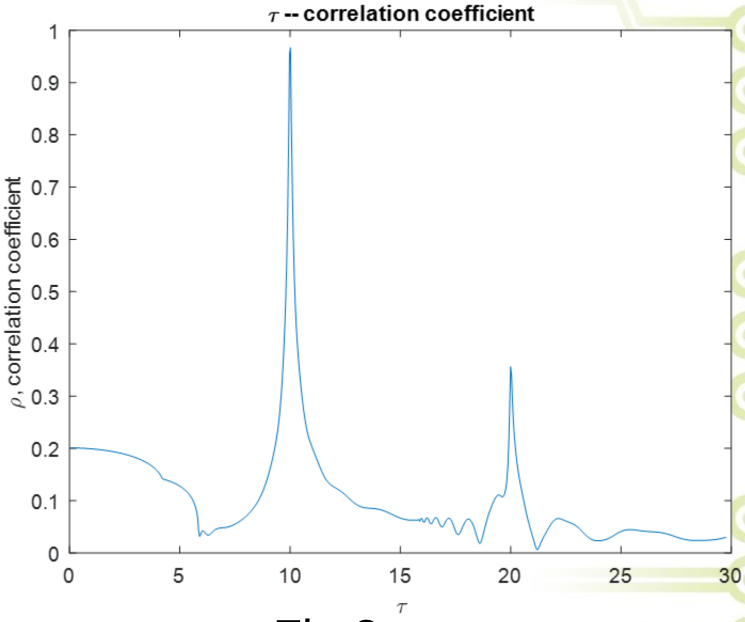


Fig.2

Basic Idea of the Proposed Algorithm

- In this case, Fig.2 can be used to determine a rough range of $fcut_i$ since two peaks are separable and clear. Nevertheless, the accuracy of $fcut_i$ is affected due to assumption of no reflection and damping.
- The empirical value of $fcut_i$'s rough range is:

SNR	eta=0.1	eta=0.3	eta=0.5
2dB	$P(\Delta fcut < 5hz) = 0.95$	$P(\Delta fcut < 10hz) = 0.94$	$P(\Delta fcut < 20hz) = 0.93$
10dB	$P(\Delta fcut < 5hz) = 0.97$	$P(\Delta fcut < 10hz) = 0.96$	$P(\Delta fcut < 20hz) = 0.96$

- Therefore, we can search over the rough range of $fcut_i$ and η_i, and for each fixed $fcut_i$ and η_i, estimate direct path and reflection path's amplitude a_i and a_{i+imax} by solving the following convex problem:

$$[a_i^*, a_{i+imax}^*] = \arg\ \min_{a_i, a_{i+imax}} (a_i \phi_0(f_\tau, \tau) + a_{i+imax}\phi_1(f_\tau, \tau) - m(f_\tau))$$

- Interference from smaller mode also affects $fcut's$ accuracy. Therefore, successive cancellation is used to increase the accuracy of subsequent modes.
- Step1, shifting the window through the signal, stops when a peak is detected.
- Step2, search $f\hat{cut}_1$ around the peak, and $\hat{\eta}_1$ over [0,1], generate a basis window sequence (BWS) start at $f\hat{cut}_1$, and estimate a_1 and a_3 by minimizing the mismatch between the BWS and the signal.
- Step3, regenerate the mode's response, and subtract it from the signal to reduce the influence to subsequent modes.

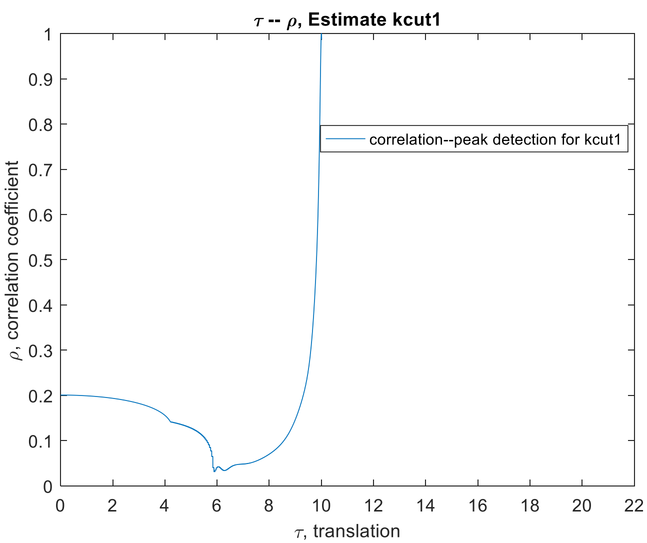


Fig.3 step 1

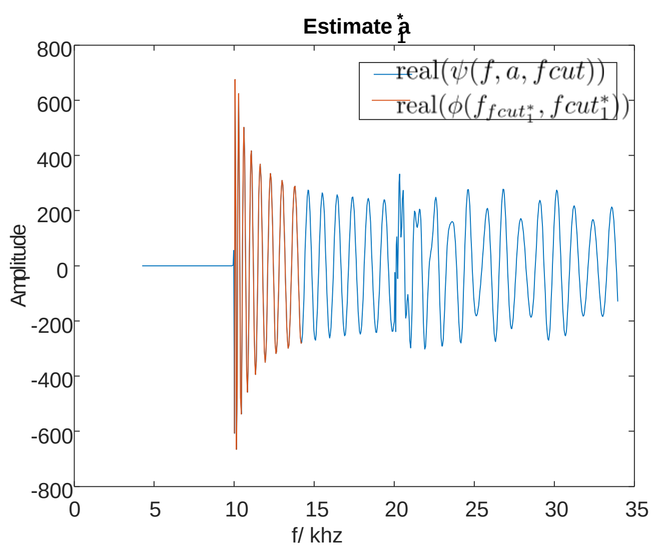


Fig.4 step 2

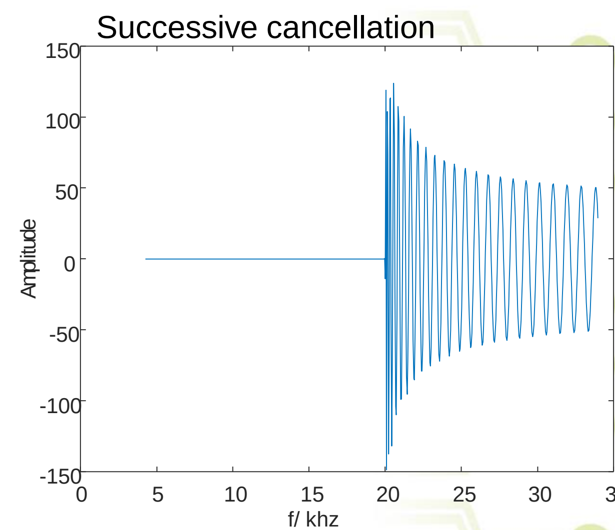


Fig.5 step 3

- Repeat these steps on the resulting signal to estimate $\hat{fcut_2}$, $\hat{\eta_2}$, $\hat{a}_2$ and $\hat{a}_4$
- Step1. Shift the window through the signal in Fig.4, stops when a peak is detected . (Fig.6 shows the improvement by using successive cancellation) .

- Step2, search $\hat{fcut_2}$ around the peak, and $\hat{\eta_2}$ over [0,1], generate a BWS start at $\hat{fcut_2}$, and estimate $\hat{a}_2$ and $\hat{a}_4$ by minimizing the mismatch between the BWS and the signal.

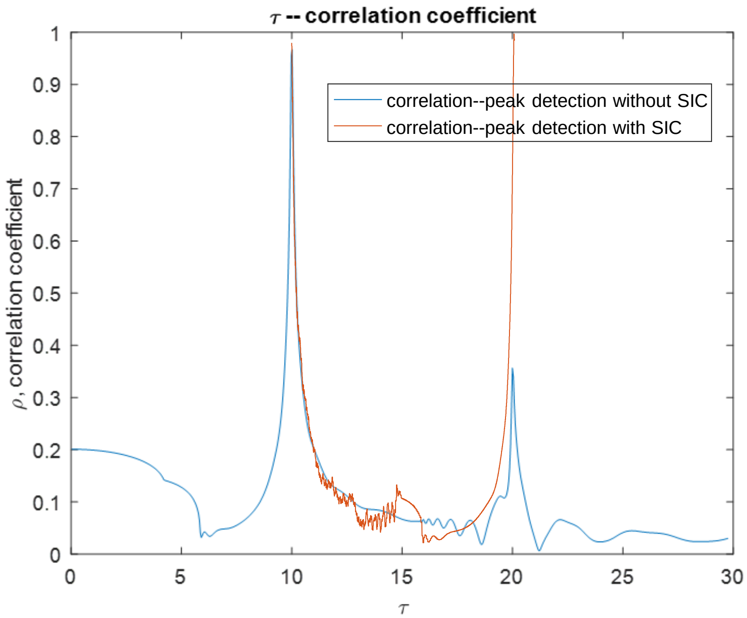


Fig.6

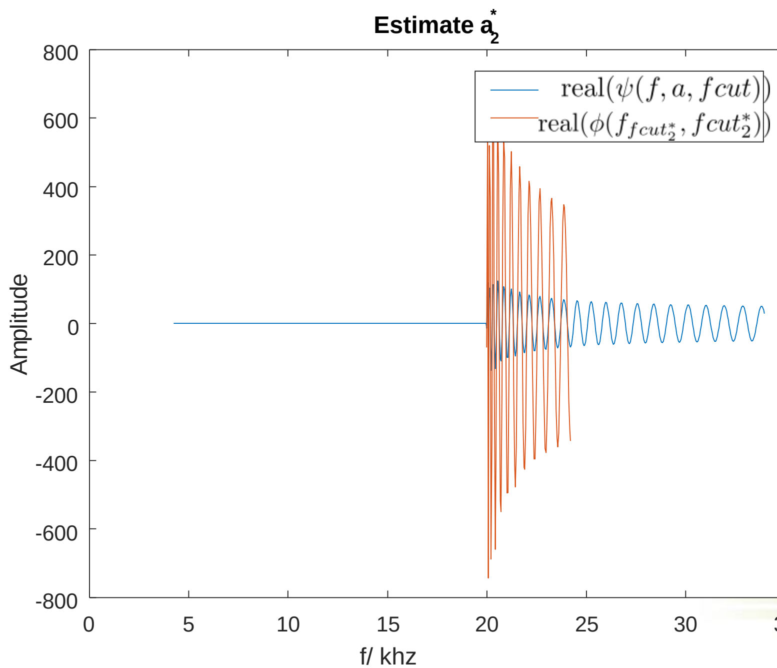


Fig.7

Proposed Algorithm's Procedure

for i=1: N

init $p_i = fcut_{i-1} + N_w \Delta f\ (fcut_1 = 0),\ c = [\,],$

Narrow $fcut_i's$ search space

while $p_i < f_{max}$

$f_{idx} = p_i : \Delta f\ : p_i + N_w \Delta f,\ seq_1 = \phi_0(f_{idx},\ p_i),\ seq_2 = m(f_{idx})$

$c = [c, corr(seq_1, seq_2)]$

$[flag, k] = peak(c)$

if $flag{=}{=}1$

$p_i = fcut_{i-1} + N_w \Delta f + k \Delta p$

else

$p_i = p_i + \Delta p$

end if

end while

Search $fcut_i$ and η_i

for $l = 1 : 1 : 2L$

$x_l = p_i + (l - L)\Delta p$

$f_{idx} = x_l : \Delta f\ : x_l + N_w \Delta f,$

$q = 1 : 1 : Q$

$e_{l,q} = \frac{q}{Q} + e_0$

Estimate a_i and a_{i+imax}

$[a_{l,q,0}, a_{l,q,1}] = \arg\min_{a_i, a_{i+imax}} (a_i \phi_0(f_{idx},\ x_i) + a_{i+imax}\phi_1(f_{idx},\ x_i)) \exp(-\frac{2e_{l,q}}{\sqrt{1-\frac{x_l}{f_{idx}}}}) - m(f_{idx})$

$seq_1 = (a_{l,q,0}\phi_0(f_{idx},\ x_i) + a_{l,q,1}\phi_1(f_{idx},\ x_i)) \exp(-\frac{2e_{l,q}}{\sqrt{1-\frac{x_l}{f_{idx}}}})$

$c'_{l,q} = [c'_{l,q}, corr(seq_1, seq_2)]$

end for

end for

Complete parameter estimation of i_{th} mode.

$[l, q] = max\ c', \hat{\eta}_i = e_{l,q}, \hat{fcut}_i = x_l, \hat{a}_i = a_{l,q,0}, \hat{a}_{i+imax} = a_{l,q,1}$

Cancel its response from the measurements.

$m = m - \hat{a}_i \phi_0(f_n,\ x_i) + \hat{a}_{i+imax}\phi_1(f_n,\ x_i)) \exp(-\frac{2\hat{\eta}_i}{\sqrt{1-\frac{\hat{fcut}}{f_n}}})$

Do it iteratively

end for

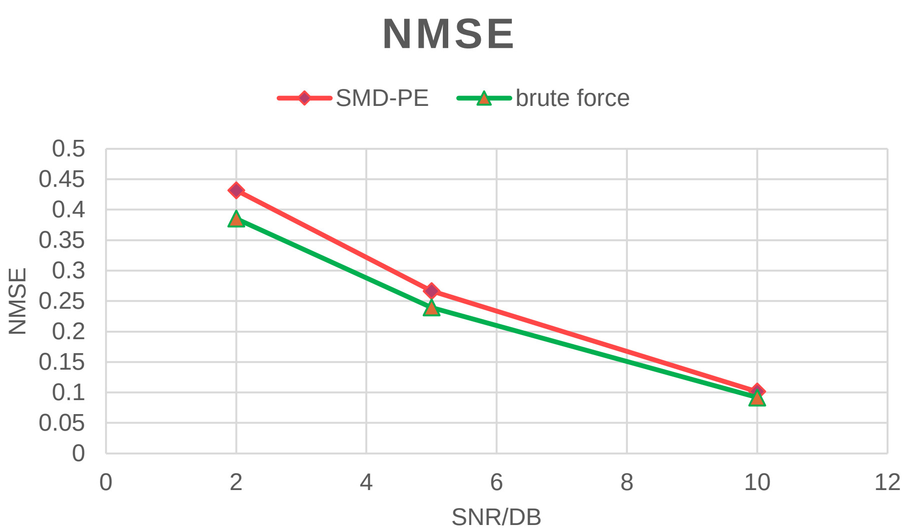
Simulation Setup

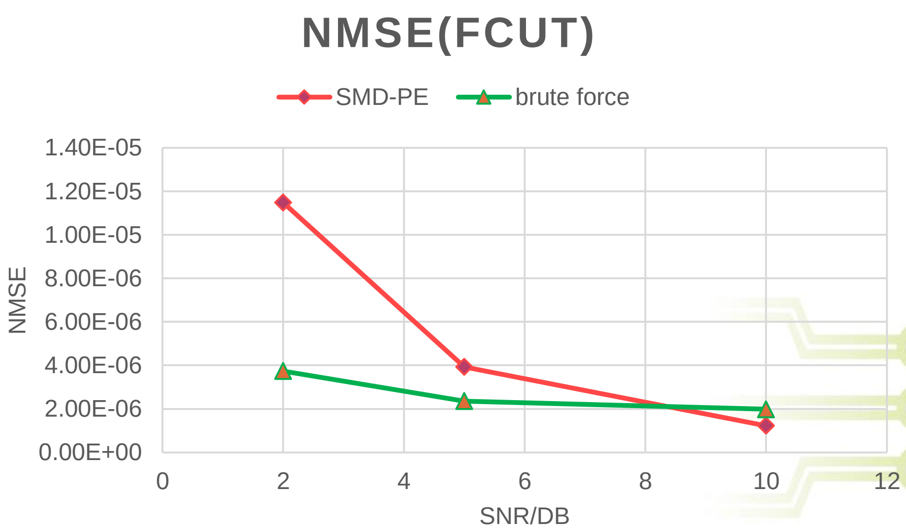
- **Speed:** $c = 340m/s$
- **Pipe radius:** $R = 0.02\,m$
- **Pipe length:** $z = 1.9\,m$
- **Number of Mode:** $N = 4$
- **Window Length:** $Nw = 300$
- **Frequency:** $f_n = 10 : 10 : 20e3\,hz$
- **Damping:** η **around** 0.5
- **Assumption:** $\min|fcut_i - fcut_i + 1| \leq N_w \Delta f = 3k\,hz$

Baseline: Brute-force Algorithm

- Brute-force Algorithm:
  - Use matlab function “fmincon” to find N local optimums with N random initial points
  - The parameter is given by the best one of the N local optimums
- Motivation of considering brute-force algorithm:
  - The brute-force algorithm is expected to achieve a performance close to the global minimum. Therefore, it can serve as a baseline for the proposed algorithm later.
  - The brute-force algorithm can be used as a benchmark to study the impact of different factors, such as equalization method, damping term, reflection term on the performance

Simulation Results

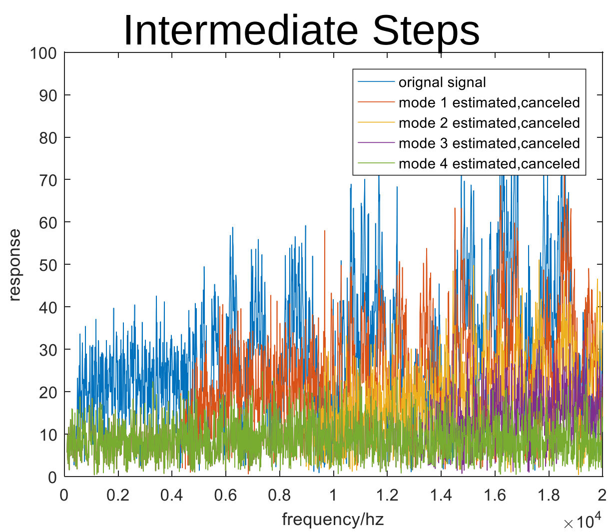


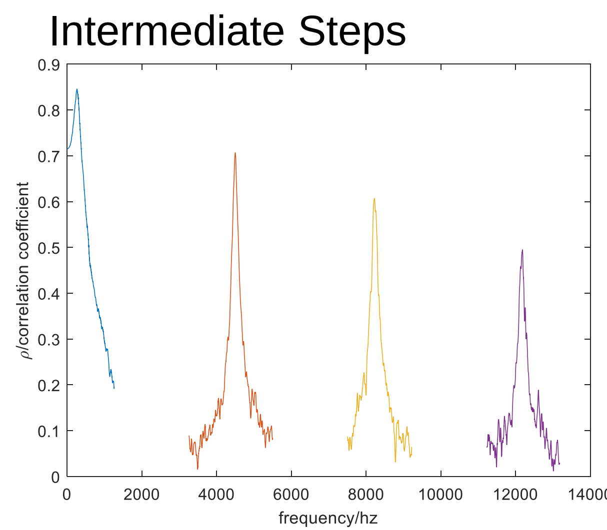


- SMD-PE achieves similar performance as brute force method.
- The deviations of $fcut$ of both methods are very small
- The SMD-PE has lower complexity than the brute-force (CPU time: around 1 minute versus 30 minutes).
- Conclusion: The SMD-PE is a reliable and low complexity method when the model is accurate and the parameters are in the operating range.

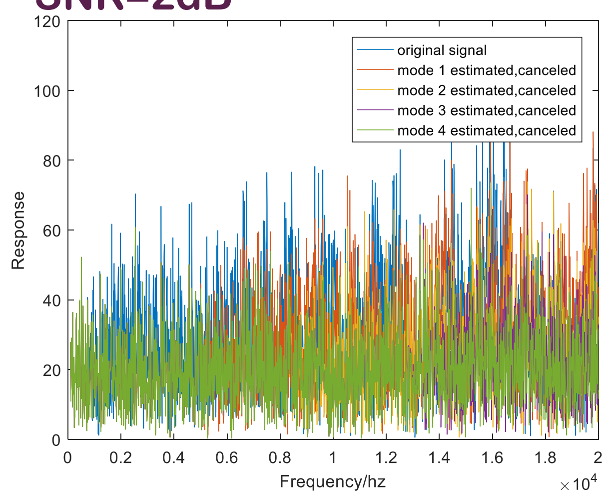
Simulation Results

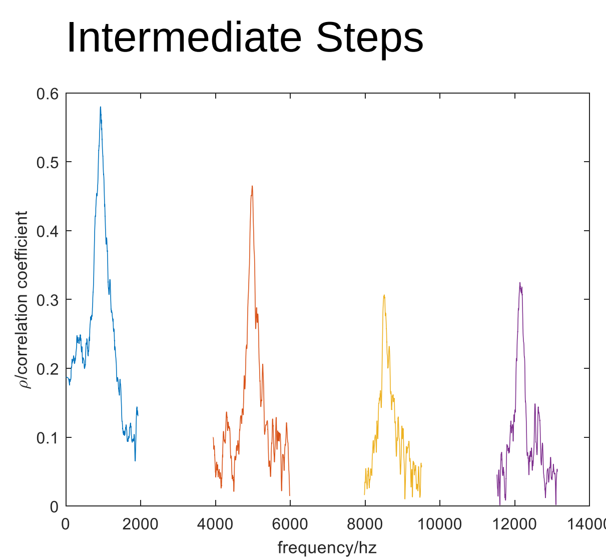
- **SNR=10dB**





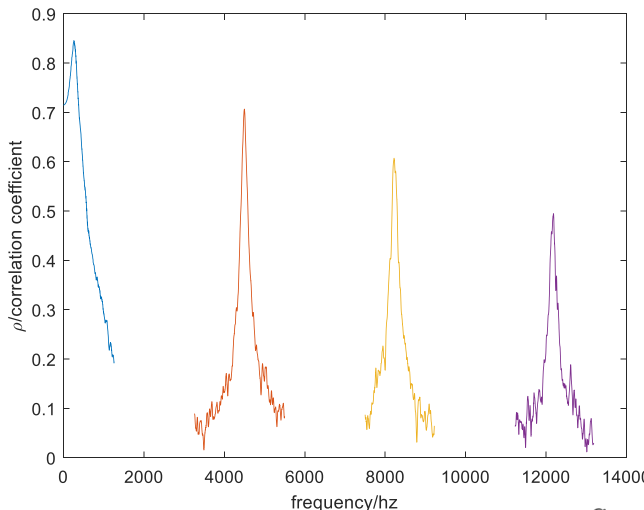
- **SNR=2dB**



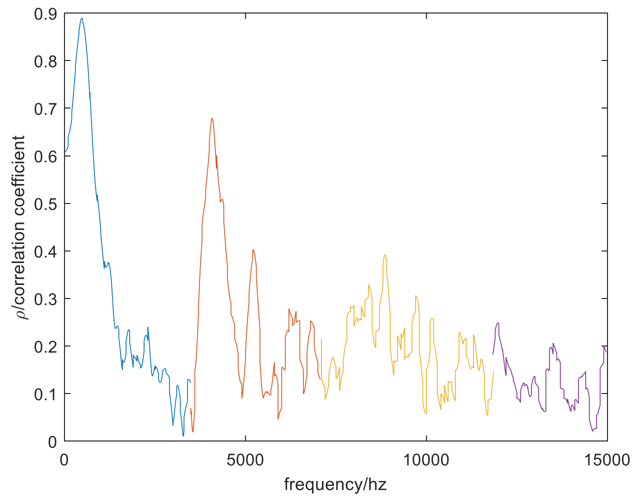


Impact of Sampling Rate/SNR

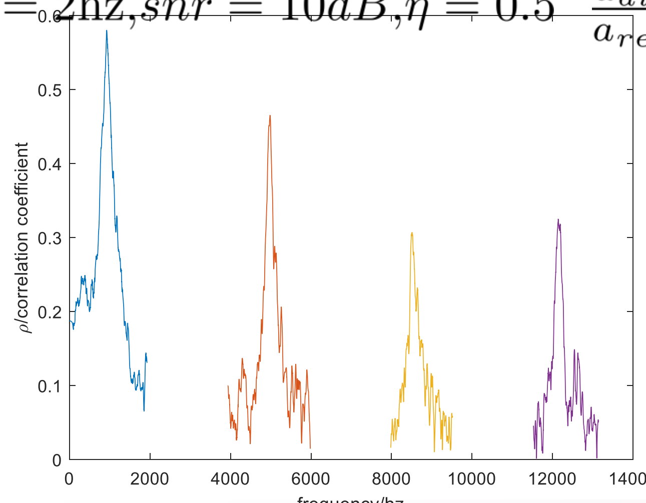
$\Delta f = 10\text{hz}, snr = 10dB, \eta = 0.5 \; \frac{a_{direct}}{a_{reflec}} \approx 0.3$



$\Delta f = 100\text{hz}, snr = 10dB, \eta = 0.5 \; \frac{a_{direct}}{a_{reflec}} \approx 0.$



$\Delta f = 2\text{hz}, snr = 10dB, \eta = 0.5 \; \frac{a_{direct}}{a_{reflec}} \approx 0.3$



Remark:

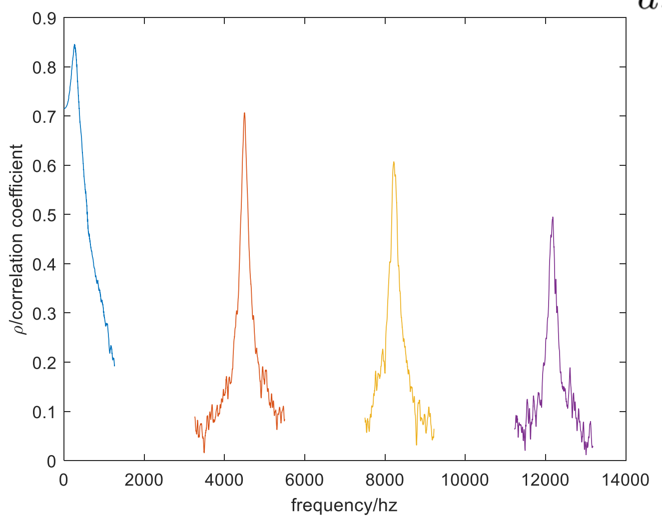
Low sampling rate, low SNR, large damping term will deteriorate the SMD-PE's performance.

Reflection term has little impact on SMD-PE's performance.

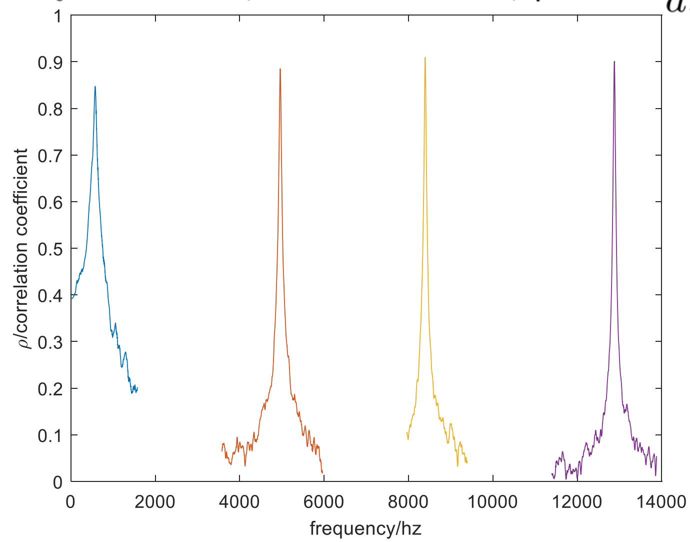
$\frac{a_{direct}}{a_{reflec}}$ represents the direct path and reflection path's strength ratio

Impact of Damping term

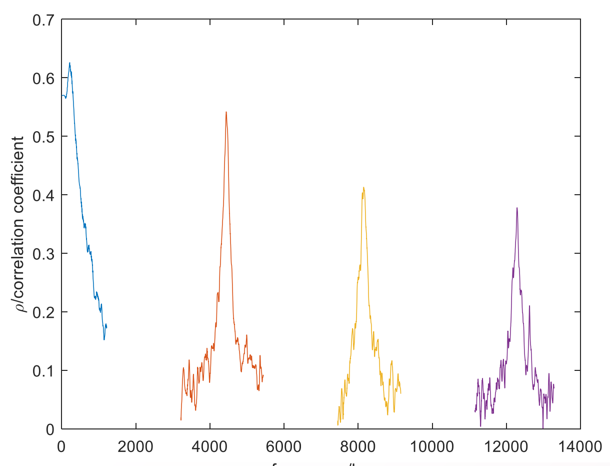
$$\Delta f = 10\text{hz}, snr = 10dB, \eta = 0.5 \quad \frac{a_{direct}}{a_{reflec}} \approx 0.3$$



$$\Delta f = 10\text{hz}, snr = 10dB, \eta = 0 \quad \frac{a_{direct}}{a_{reflec}} \approx 0.3$$



$$\Delta f = 10\text{hz}, snr = 10dB, \eta = 0.5 \quad \frac{a_{direct}}{a_{reflec}} \approx 1$$



Remark:

Low sampling rate, low SNR, large damping term will deteriorate the SMD-PE's performance.

Reflection term has little impact on SMD-PE's performance.

$\frac{a_{direct}}{a_{reflec}}$ represents the direct path and reflection path's strength ratio

Result (Air Channel Measurements)

- **Theoretical value:**

$$fcut = [0, 4981, 8263, 10367, 11366, 14387, 14424, 17358, 18144, 18981, 20295]$$

- **Estimated value:**

Method	Estimated value	NMSE
Brute Force	$[185, 4993, 10343, 13610]$	0.69
Successive Detection	$[474, 5022, 10477, 14609]$	0.83

- **Speed:** $c = 340m/s$
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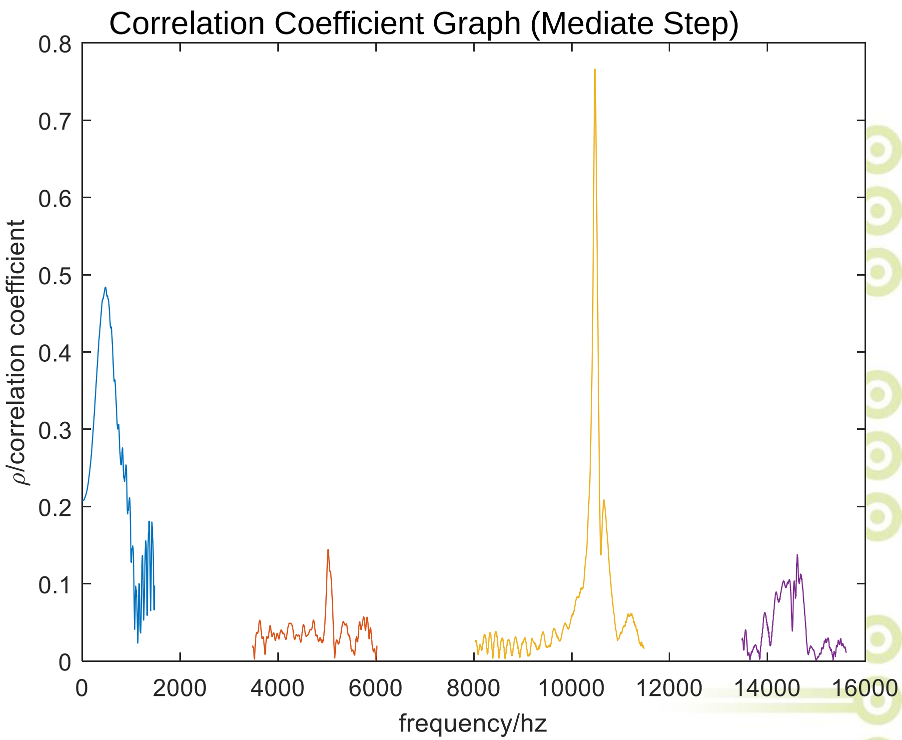
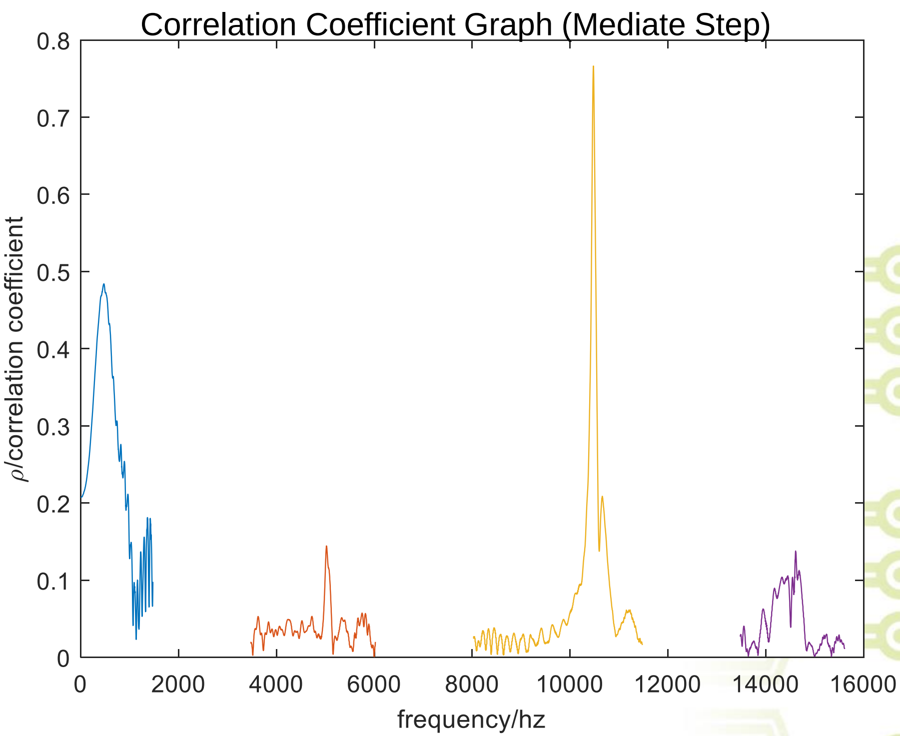


Result (Air Channel Measurements)

- The performance of both algorithms degrades significantly due to
  - Model mismatch
  - Inaccurate speaker response

- In the SMD-PE, we can still observe two clear peaks corresponding to two dominate modes



Conclusion and Future Plans

- Conclusion:
- 1. The SMD-PE is a reliable and low complexity method when the model is accurate and the parameters (e.g., SNR and sampling rate) are in the operating range.
- 2. Apply SMD-PE to air channel measurements, we can observe two clear peaks corresponding to two dominate modes
- 3. The results suggest that the model is not completely consistent with the measurements and/or the speaker response is inaccurate.
- Future plans
  - Explore