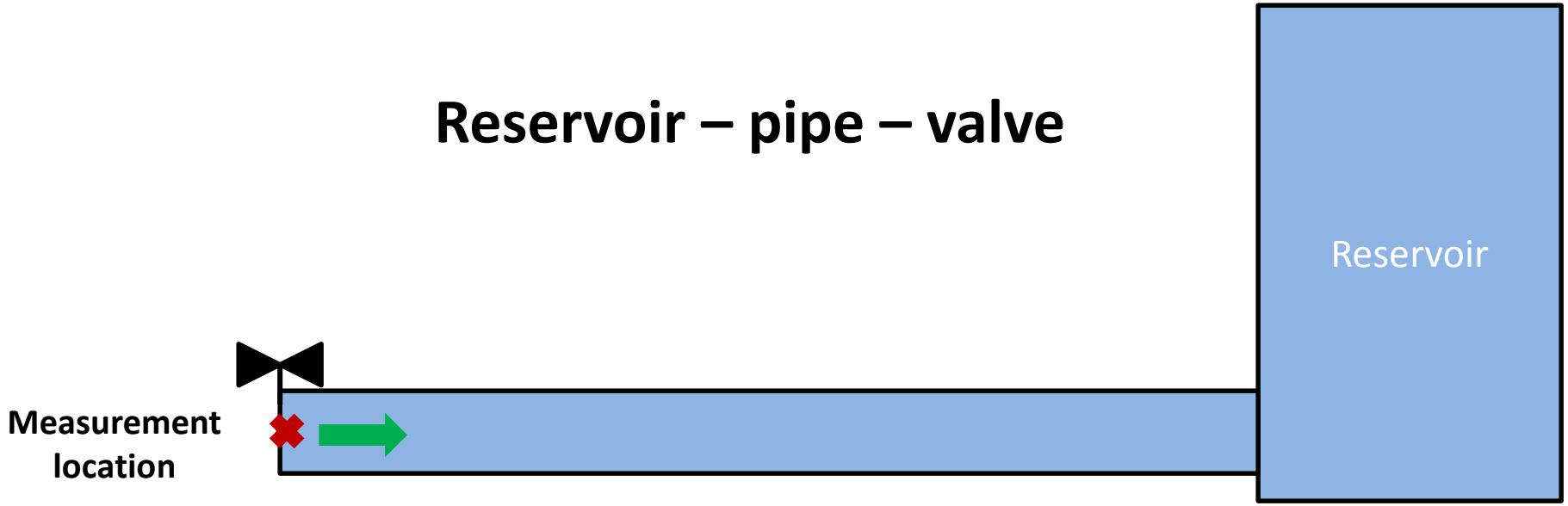
Leak-Wave Interaction

Reservoir – pipe – valve

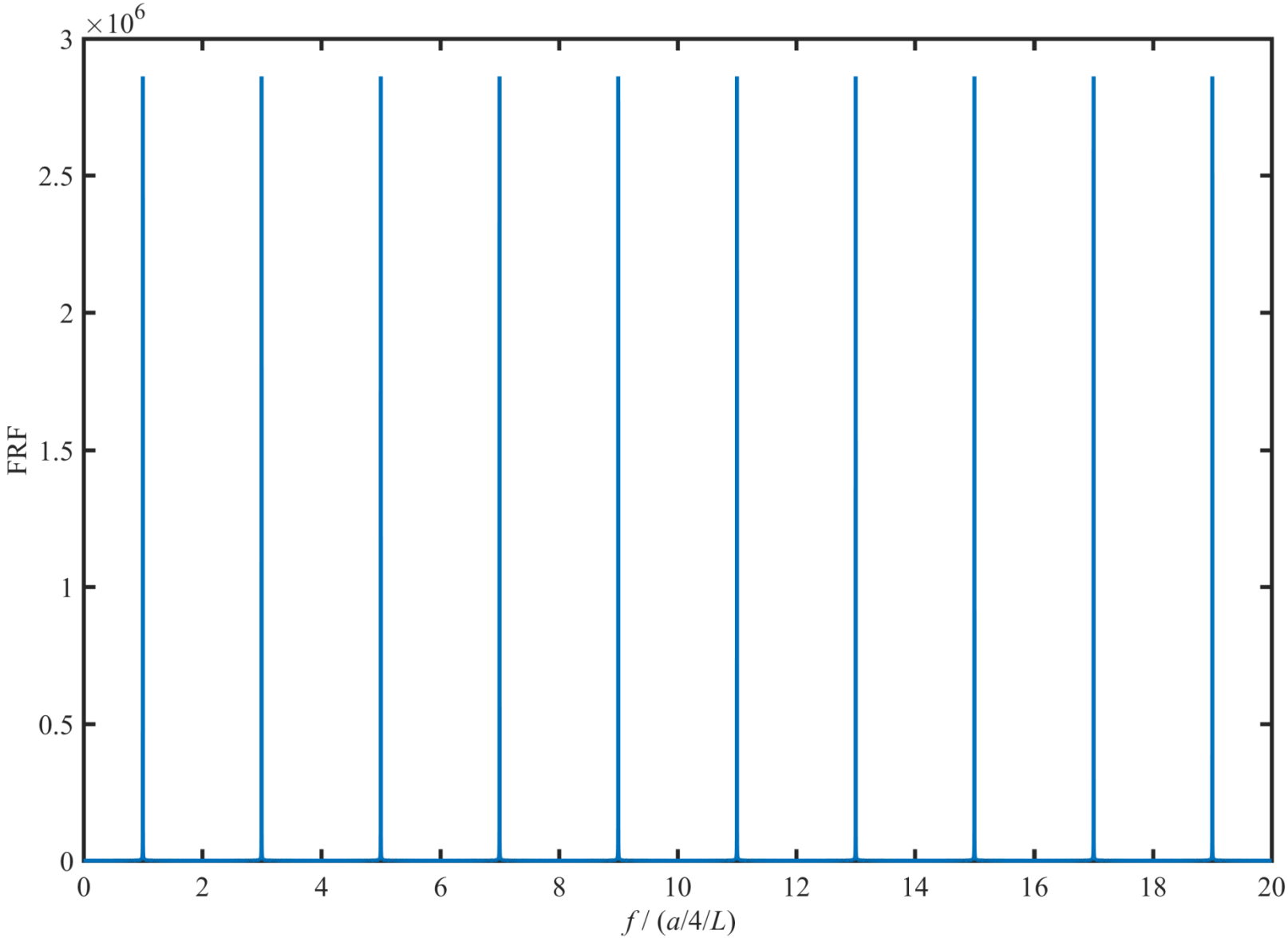


Pressure time signal

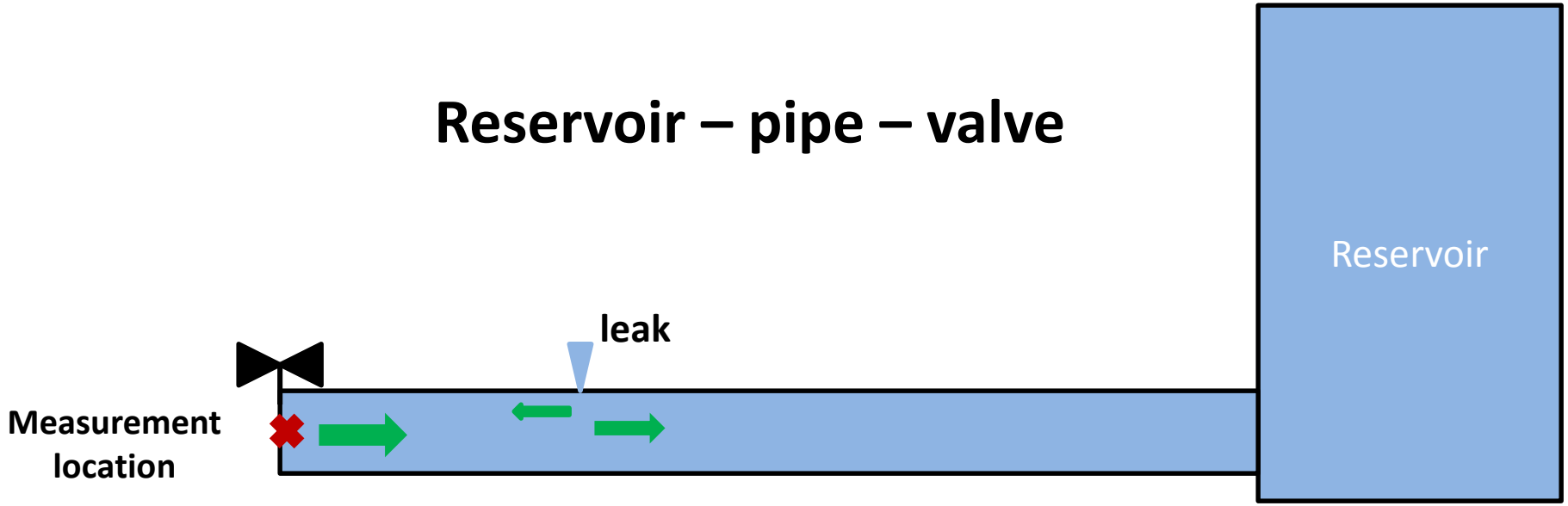
Fourier Transform

Frequency Response Function

Frequency Response Function (FRF)



Reservoir – pipe – valve

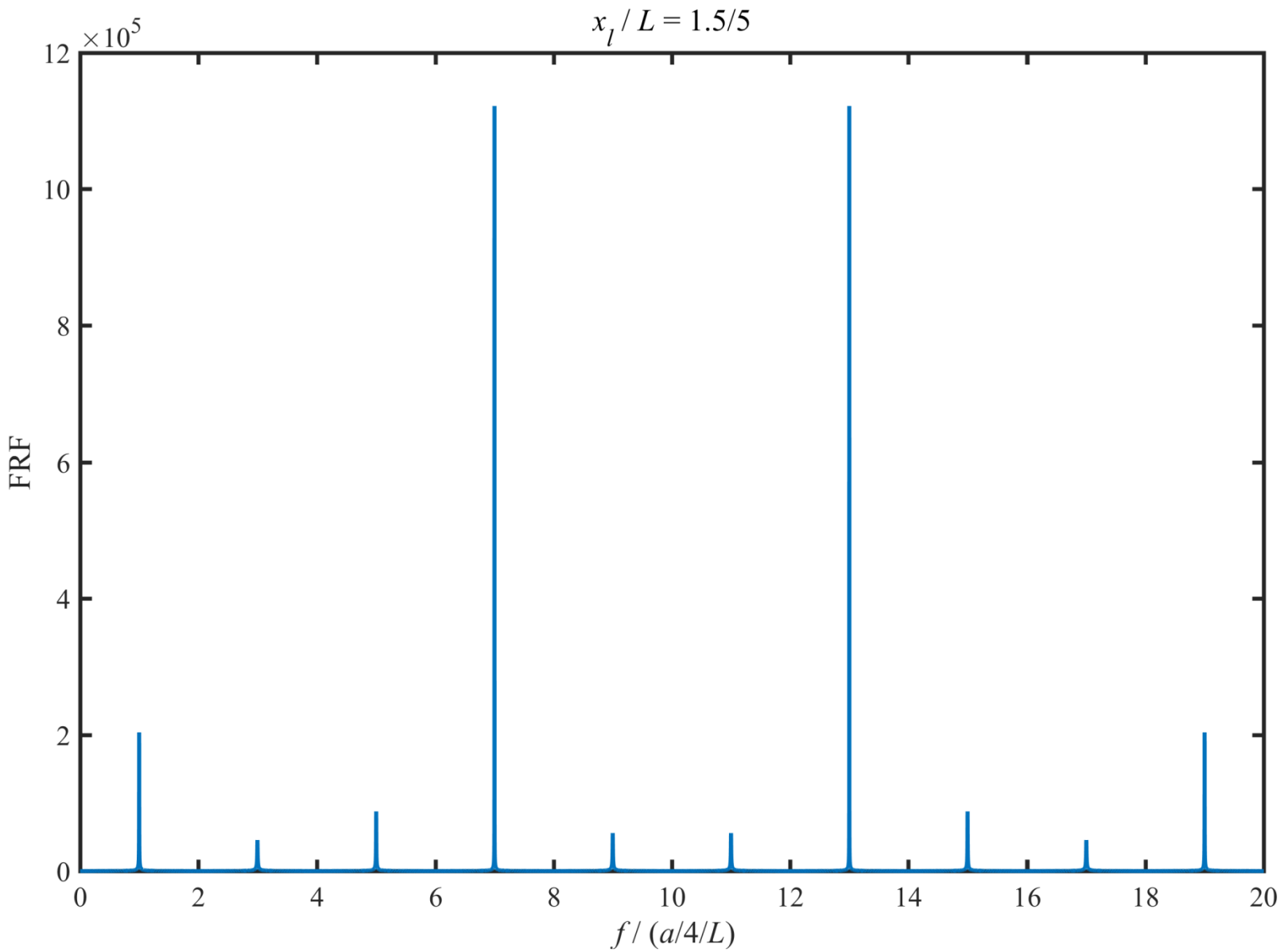


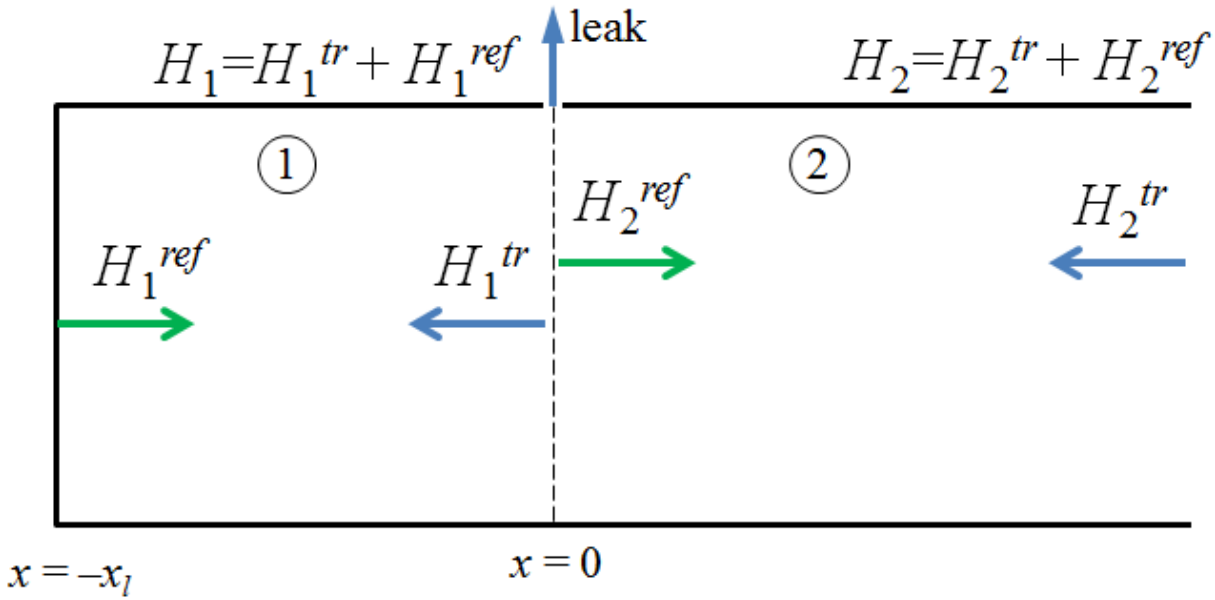
Pressure time signal

Fourier Transform

Frequency Response Function

Frequency Response Function (FRF)





$$\boxed{\left|\frac{H_2^{ref}}{H_0}\right|^2 = \frac{1 - 2sZ(1 - sZ/2)\cos^2(kx_L)}{1 + 2sZ(1 + sZ/2)\cos^2(kx_L)}}$$

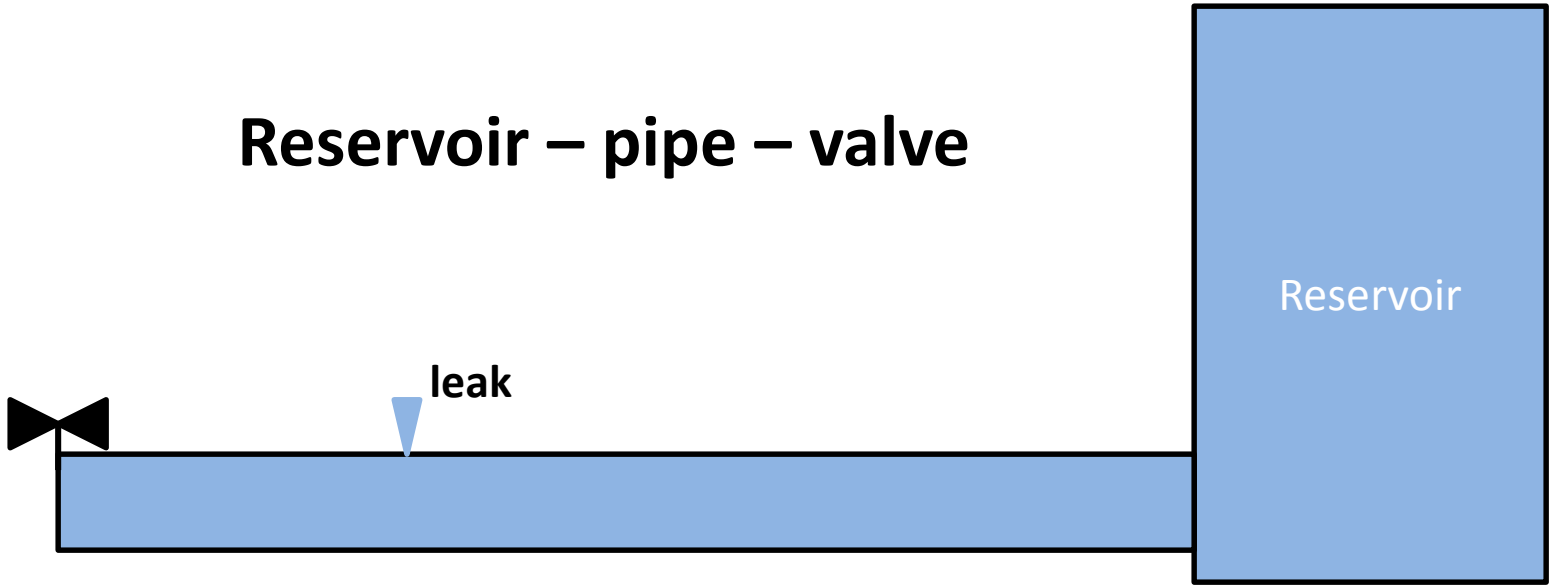
Maximum reflection

$$\cos(kx_l) = 0 \quad \Rightarrow \quad w_n^R(x_l) = 2\pi\left[(2n-1)\frac{a}{4x_l}\right]$$

Minimum reflection

$$\sin(kx_l) = 0 \quad \Rightarrow \quad w_n^T(x_l) = 2\pi\left[2(n-1)\frac{a}{4x_l}\right]$$

Reservoir – pipe – valve



Resonant frequencies:

$$\cos\left(k_m L\right) = 0 \quad \Rightarrow \quad w_m = a k_m = 2\pi\left[\left(2m-1\right)\frac{a}{4L}\right] \quad ; \quad m \in \mathbb{N}^+$$

Maximum reflection

$$w_m = w_n^R\left(x_l\right) \quad \Rightarrow \quad \frac{x_l}{L} = \frac{2n-1}{2m-1}$$

Minimum reflection

$$w_m = w_n^T\left(x_l\right) \quad \Rightarrow \quad \frac{x_l}{L} = \frac{2n}{2m-1}$$

Transfer Matrix Method

$$\textbf{FRF} = \boxed{\left|\frac{M_{u-x}^{12}}{M_{u-d}^{22}}\right|^2 = \frac{\left(Z_c^0\right)^2\left[\sin^2\left(kx_M\right)+\left(Z_c^0/Z_L\right)^2\sin^2\left(kl_1\right)\sin^2\left(k\left(x_M-l_1\right)\right)\right]}{\cos^2\left(kL\right)+\left(Z_c^0/Z_L\right)^2\sin^2\left(kl_1\right)\cos^2\left(kl_2\right)};\quad \left(\text{e.g. } x_M = L\right)}$$

Maximum reflection

$$w_m = w_n^R\left(x_l\right) \quad \Rightarrow \quad \frac{x_l}{L} = \frac{2n-1}{2m-1}$$

FRF = Z_L

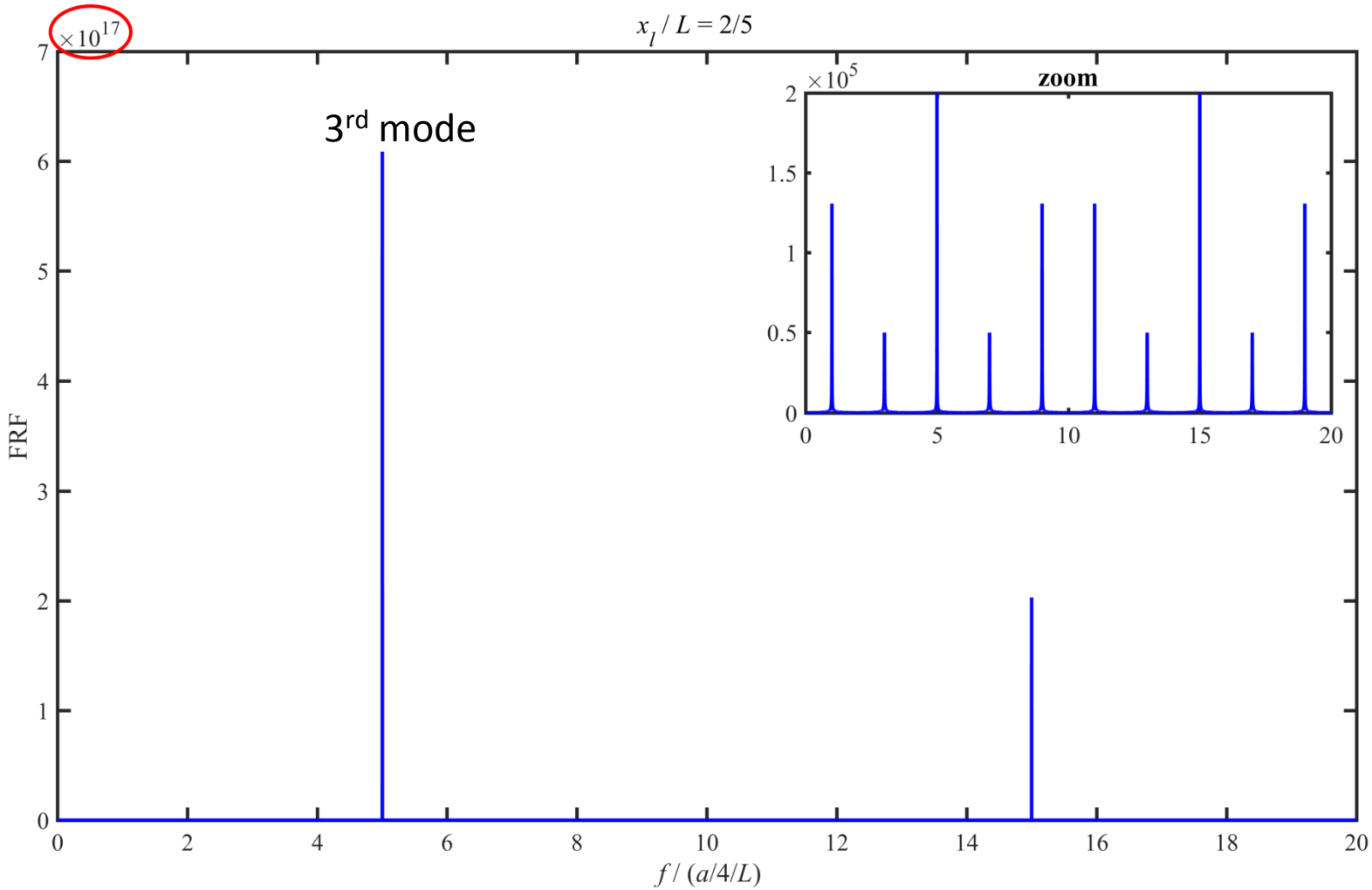
Minimum reflection

$$w_m = w_n^T\left(x_l\right) \quad \Rightarrow \quad \frac{x_l}{L} = \frac{2n}{2m-1}$$

FRF = ∞

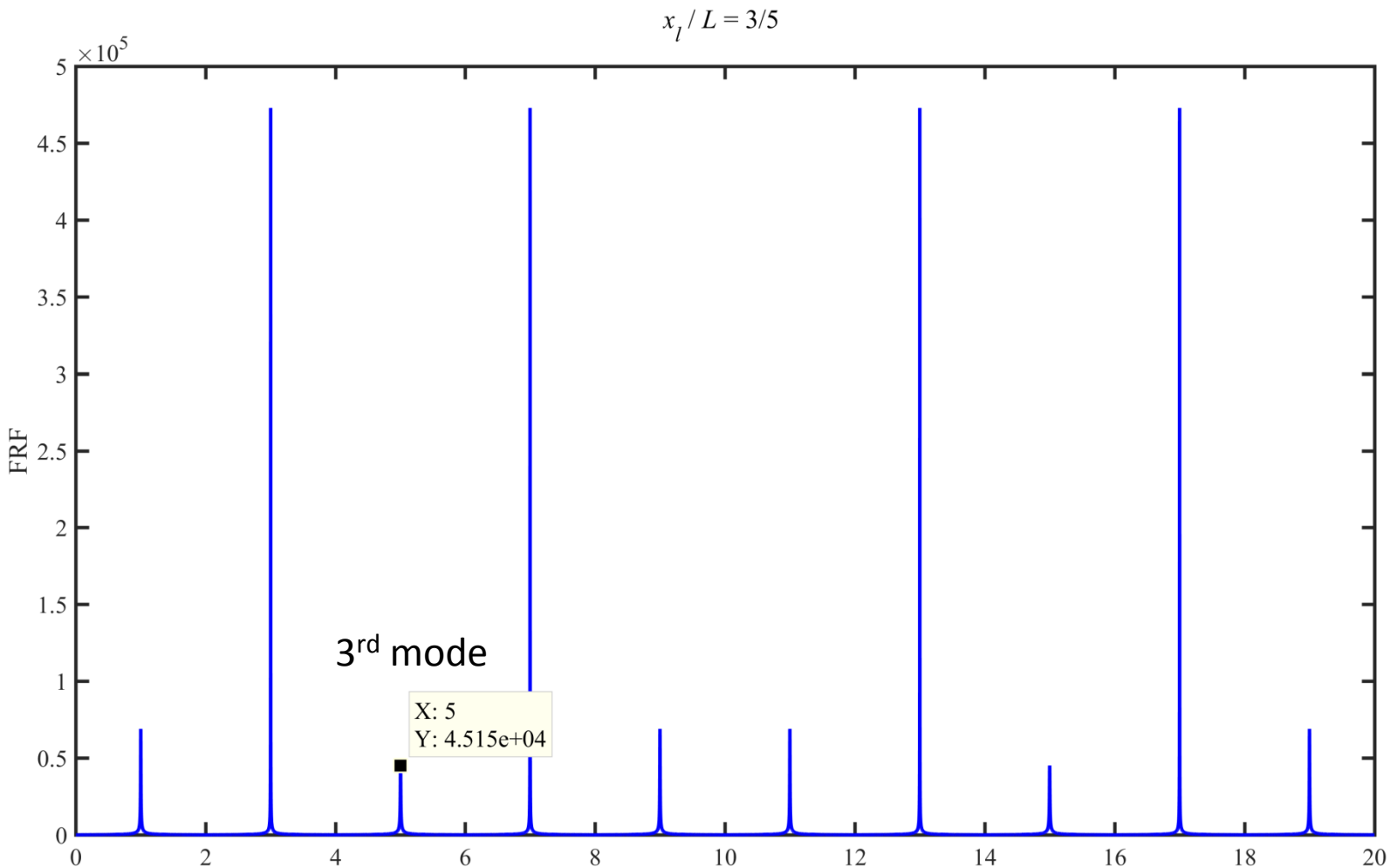
Minimum reflection

FRF = ∞



Maximum reflection

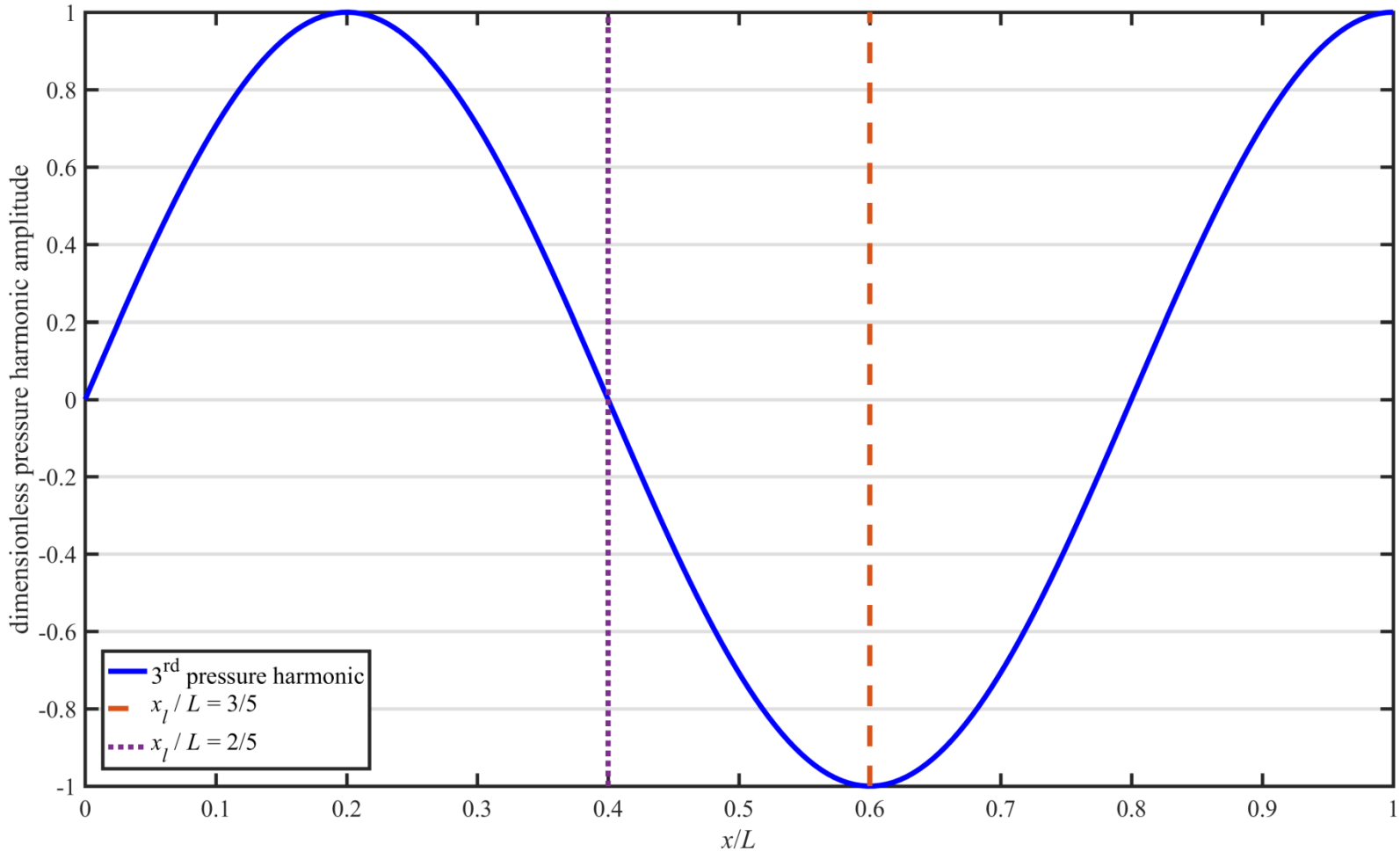
FRF = Z_L

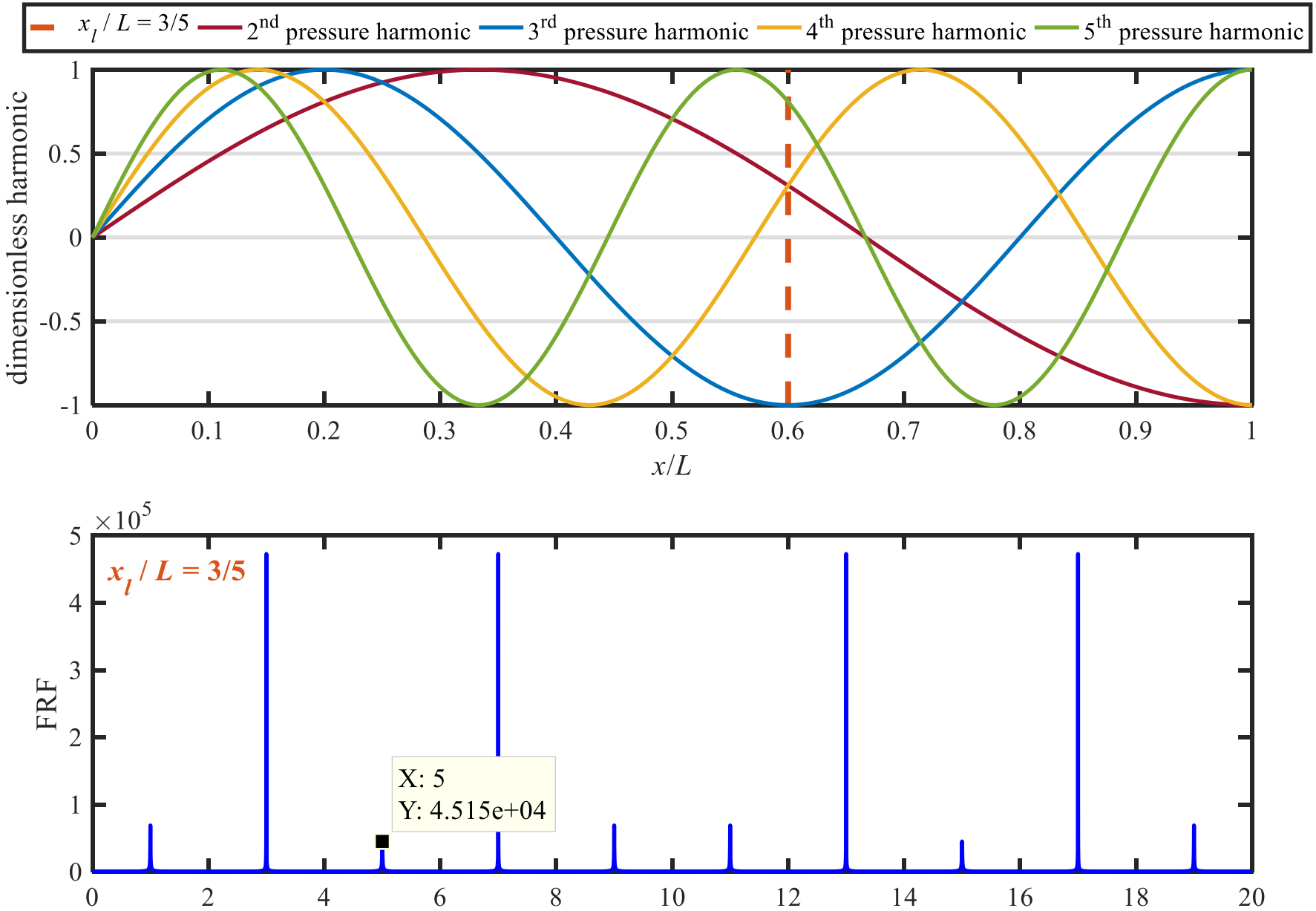


3rd mode pressure harmonic

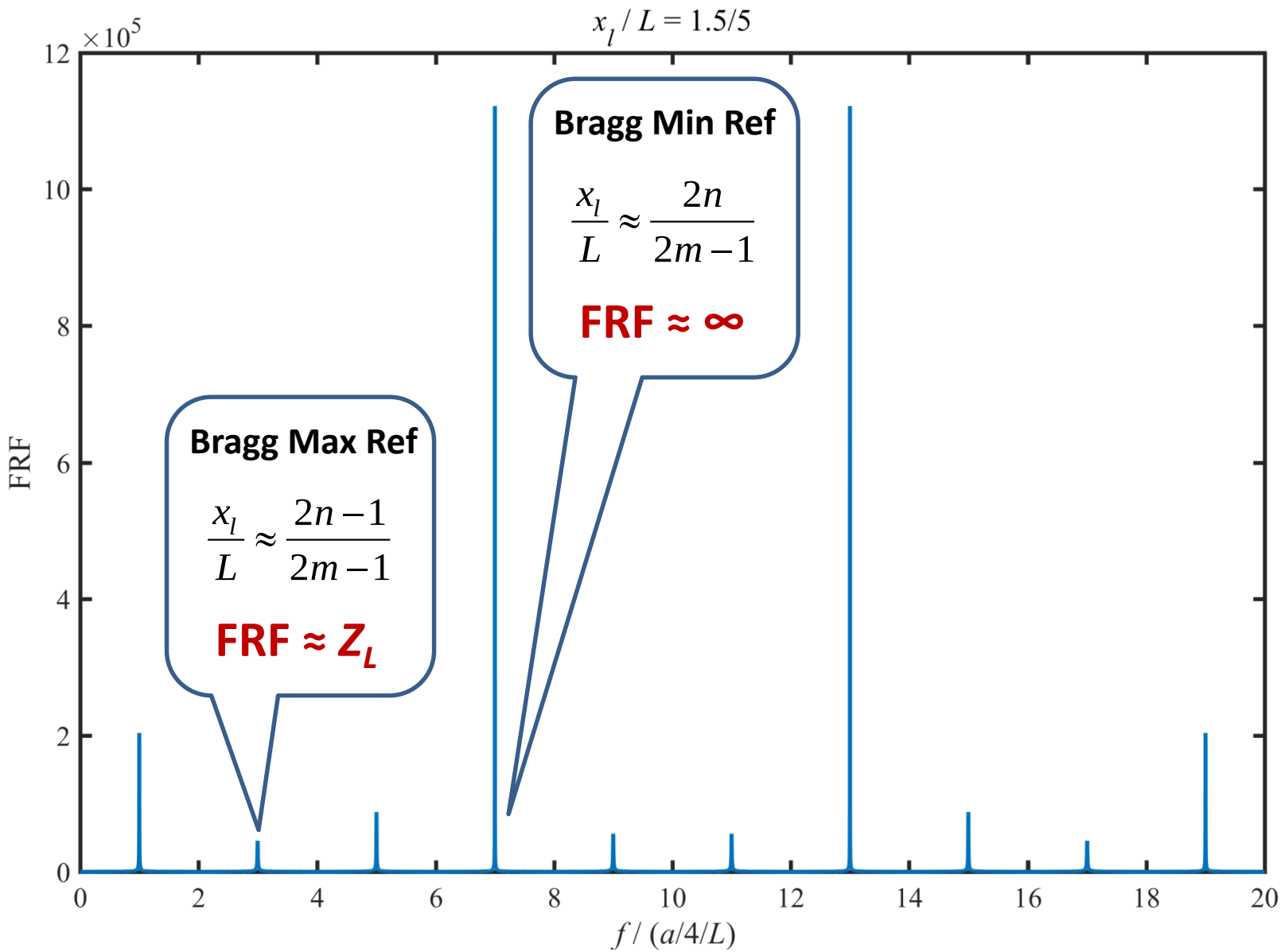
Reservoir BC

Valve BC




x_l / L = 3/5
2nd pressure harmonic
3rd pressure harmonic
4th pressure harmonic
5th pressure harmonic
dimensionless harmonic
1
0.5
0
-0.5
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x/L
$\times 10^5$
5
4
3
2
1
0
FRF
x_l / L = 3/5
X: 5
Y: 4.515e+04
0
2
4
6
8
10
12
14
16
18
20

General case: Friction + non-special leak location



THANK YOU