Leak-induced Frequency Shifts in Transient-based Frequency Domain Method for Leakage Detection

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Content

1 Introduction

Resonance frequency based leakage detection method has increasingly become a topic of research (Chaudhry 1987, Wylie and Streeter 1978, Mpesha et al. 2001, Ferrante et al. 2001, Lee et al. 2005)

However Discussion Remains Open:
Does leak shifts the resonance frequency of the FRF of the pipe?

- Lee et al. assumes that a leak in pipeline is so small that it does not shift the resonance frequency
- Mpeha et al. claims that leak would induce additional peak in FRF (in their case, leak leaking out 80% of water)
- Ferrant et al. founds that with the increasing size of the leak, the resonance frequency of the pipeline will change gradually and finally FRF of the system will be greatly changed

2 Proof of leak-induced shift

- Small size of leak will be considered for the analytical derivation
- First order Taylor expansion is used for approximation
- Resonance wavenumber is found by searching for the singularities of FRF
- Analytical expression of leak-induced shift is derived

Proof of leak-induced shift

FRF : $h_D(k) = \dfrac{-\frac{ia}{gA}\sin kl + \frac{a}{gA}\frac{Z_C}{Z_L}\sin k(l-x_l)\sin kx_l}{\cos kl + i\frac{Z_C}{Z_L}\cos k(l-x_l)\sin kx_l}$

Denominator of FRF : $f(k) = \cos kl + i\cdot\dfrac{Z_C}{Z_L}\cos k(l-x_l)\sin kx_l$

Search for singularities of FRF : $f(k_n) = f(k_n^{NL} + \Delta k^*) \approx \underbrace{f(k_n^{NL}) + \Delta k_n^* \cdot f'(k_n^{NL})} = 0$

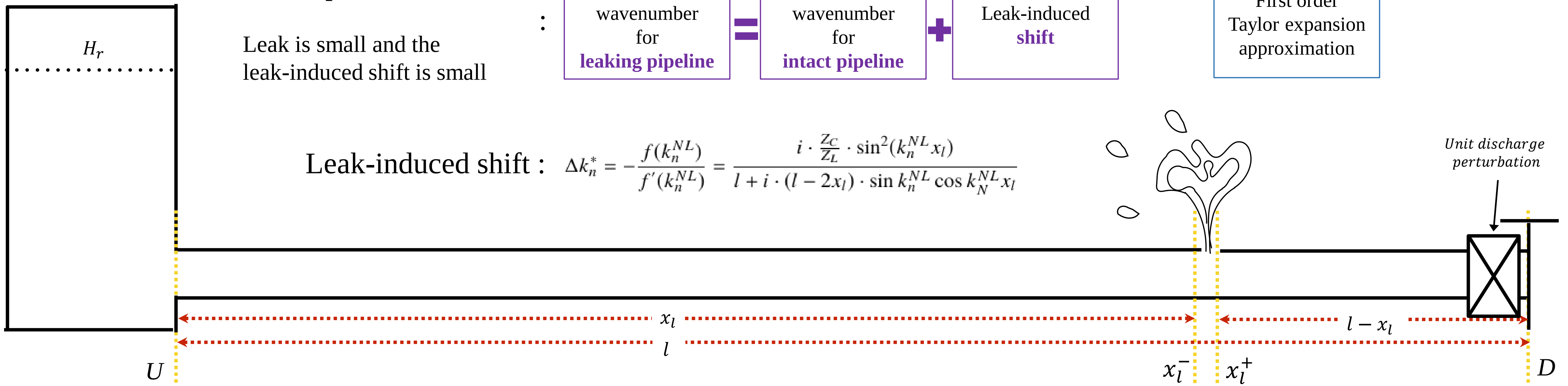
Assumption

Leak is small and the leak-induced shift is small

: Resonance wavenumber for **leaking pipeline** = Resonance wavenumber for **intact pipeline** + Leak-induced **shift**

First order Taylor expansion approximation

Leak-induced shift : $\Delta k_n^* = -\dfrac{f(k_n^{NL})}{f'(k_n^{NL})} = \dfrac{i\cdot\frac{Z_C}{Z_L}\cdot\sin^2(k_n^{NL}x_l)}{l + i\cdot(l-2x_l)\cdot\sin k_n^{NL}\cos k_N^{NL}x_l}$



Summary for Part2:

1. **Leak-induced resonance frequency shift is analytical proved to be exist**
2. **The leak-induced shift is a complex number;**

3 Numerical Verification

Two numerical methods are used for verification of the analytical derived shift expression

1. Numerical Method for searching resonance wavenumbers of FRF from theory of complex variable:

Reference: Kravanja, P. and Van Barel, M. (2007) 《Computing the zeros of analytic functions》

$$k_n' = \frac{1}{2\pi i}\oint_{C_n} \frac{k f'(k)}{f(k)} dk, \qquad n = 1, 2, 3 \ldots$$

$$\Delta k_n' = k' - k_{th}^{NL} = \frac{1}{2\pi i}\oint_{C_n} \frac{k f'(k)}{f(k)} dk - \frac{\pi}{2l}, \qquad n = 1, 2, 3 \ldots$$

where *f(k)* is the denominator of FRF :

$$f(k) = \cos kl + i \cdot \frac{Z_C}{Z_L} \cos k(l - x_l) \sin k x_l$$

2. MOC simulation for searching resonance wavenumbers:

For verification of the analytical derived shift expression

❖ **Analytical shift expression:**

$$\Delta k_n^* = -\frac{f(k_n^{NL})}{f'(k_n^{NL})} = \frac{i \cdot \frac{Z_C}{Z_L} \cdot \sin^2(k_n^{NL} x_l)}{l + i \cdot (l - 2x_l) \cdot \sin k_n^{NL} \cos k_N^{NL} x_l}$$

Notice:

1. $\frac{Z_C}{Z_L}$ **is chosen to be the parameter representing the size of the leak, the larger value of** $\frac{Z_C}{Z_L}$ **, the larger the size of the leak**

2. **In study conducted in Part 2-3, we select** $\mathbf{0 < \frac{Z_C}{Z_L} < 1.0}$ **, which covers the widest practical range in engineering situation.**

3. **Two value of** $\frac{Z_C}{Z_L}$ **is selected for verification** $\frac{Z_C}{Z_L} = \mathbf{0.1355}$ **and** $\frac{Z_C}{Z_L} = \mathbf{0.7227}$

Explanation:

Z_{ls} be within the range 30–60m;

a be of the order of 1000 m/s ;

$C_d A_l$ be within the range 0.001–0.01.

Leads to

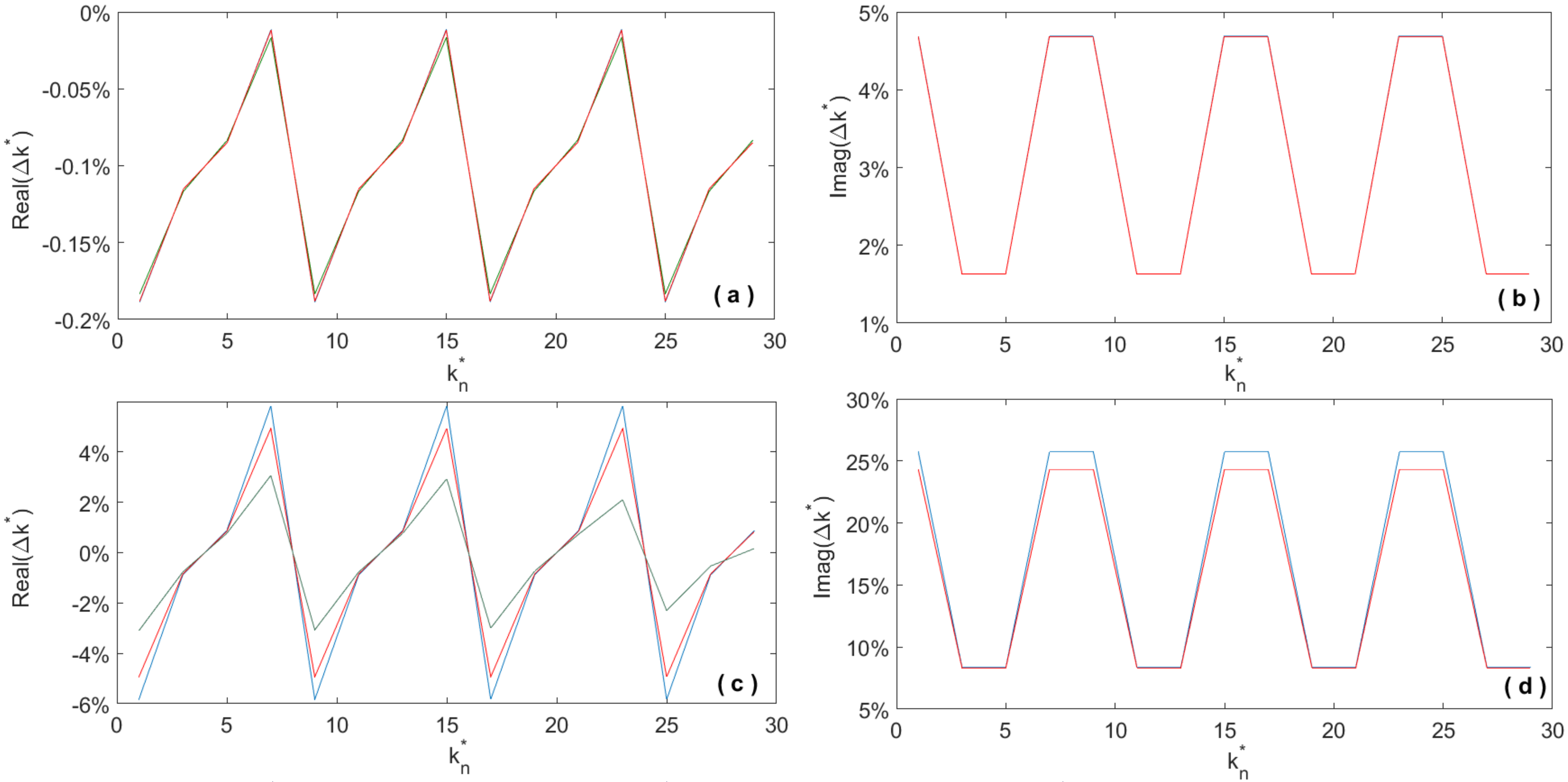
$$0 < \frac{Z_C}{Z_L} < 1.0$$

Comparison between

Analytical derived shift expression VS $\left\{ \begin{array}{c} \text{Theory of complex variable} \\ \& \\ \text{MOC simulation} \end{array} \right.$

$$\frac{Z_C}{Z_L} = 0.1355$$

$$\frac{Z_C}{Z_L} = 0.7227$$



Summary for Part3:

1. Leak-induced resonance frequency shift does exist and it is a complex number, which can be proved from both **analytical derivation** and **numerical simulation.**

2. Within the range $0 < \frac{Z_C}{Z_L} < 1.0$, the first order Taylor expansion approximation can accurately capture the leak induced resonance frequency shift.

4

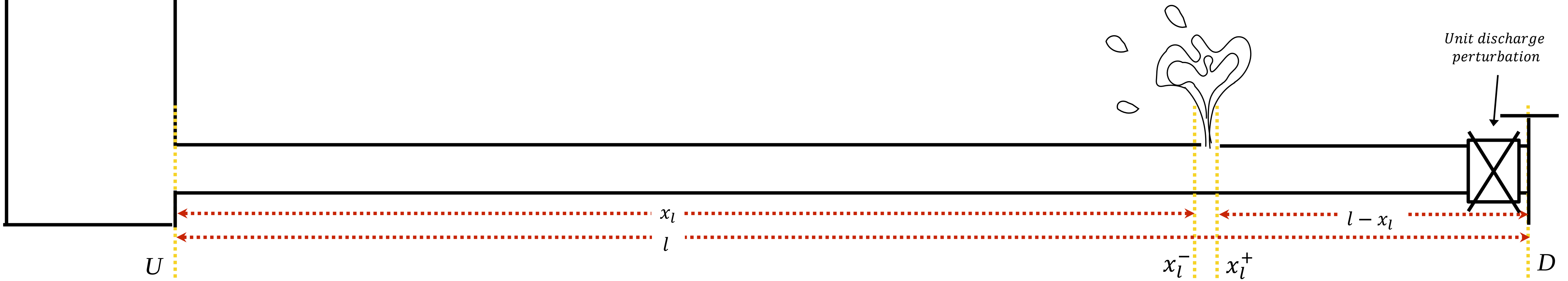
Leak size effect on shift

Leaking pipe case under normal leak size

Leak size ↑ , $\frac{Z_C}{Z_L}$ ↑. So by changing the value of $\frac{Z_C}{Z_L}$

So by changing the value of $\frac{Z_C}{Z_L}$, we can obtain FRF for different leak size cases using the function showed below:

$$h^D(k) = \frac{-\frac{ia}{gA}\sin kl + \frac{a}{gA}\frac{Z_C}{Z_L}\sin k(l - x_l)\sin kx_l}{\cos kl + i\frac{Z_C}{Z_L}\cos k(l - x_l)\sin kx_l}$$



Leaking pipe case under normal leak size

$$\begin{bmatrix} q \\ h \\ 1 \end{bmatrix}^{D} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos k(l-x_l) & -i\frac{1}{Z_C}\sin k(l-x_l) & 0 \\ -iZ_C \sin k(l-x_l) & \cos k(l-x_l) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ h \\ 1 \end{bmatrix}^{x_l^+}$$

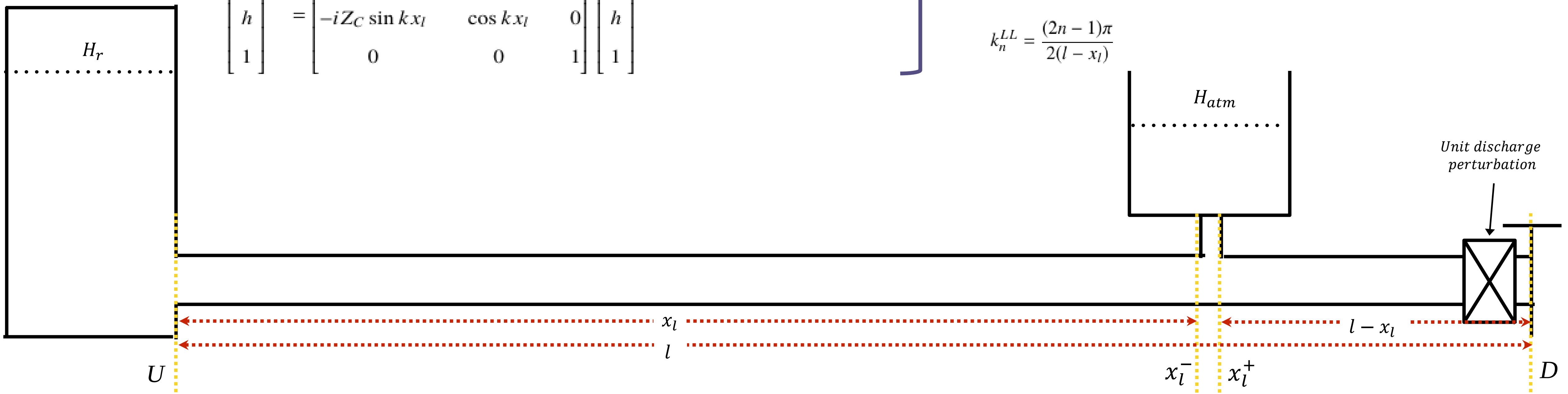
$$\begin{bmatrix} q \\ h \\ 1 \end{bmatrix}^{x_l^-} = \begin{bmatrix} \cos kx_l & -i\frac{1}{Z_C}\sin kx_l & 0 \\ -iZ_C \sin kx_l & \cos kx_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ h \\ 1 \end{bmatrix}^{U}$$

FRF for extremely large leak case :

$$h^D(k) = i \cdot Z_C \cdot \frac{\sin k(l-x_l)}{\cos k(l-x_l)}$$

Resonance wavenumber :

$$k_n^{LL} = \frac{(2n-1)\pi}{2(l-x_l)}$$

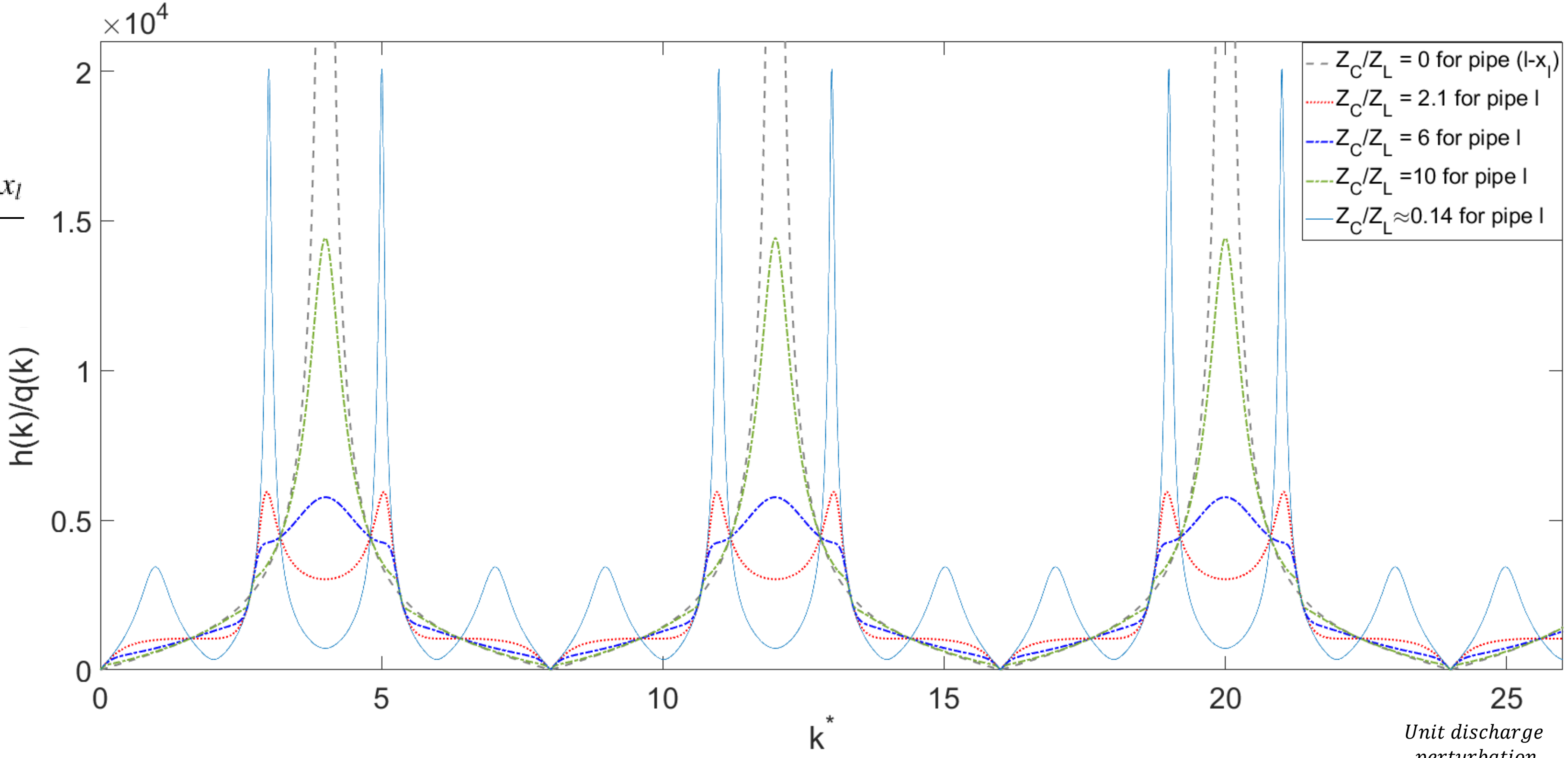


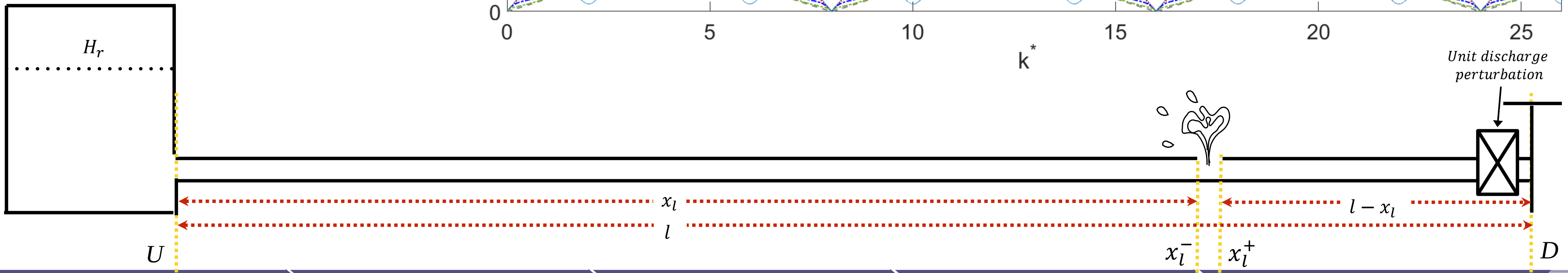
FRF for normal leak case :

$$h^D(k) = \frac{-\frac{ia}{gA}\sin kl + \frac{a}{gA}\frac{Z_C}{Z_L}\sin k(l-x_l)\sin kx_l}{\cos kl + i\frac{Z_C}{Z_L}\cos k(l-x_l)\sin kx_l}$$

FRF for extremely large leak case :

$$h^D = i \cdot Z_C \cdot \frac{\sin k(l-x_l)}{\cos k(l-x_l)}$$

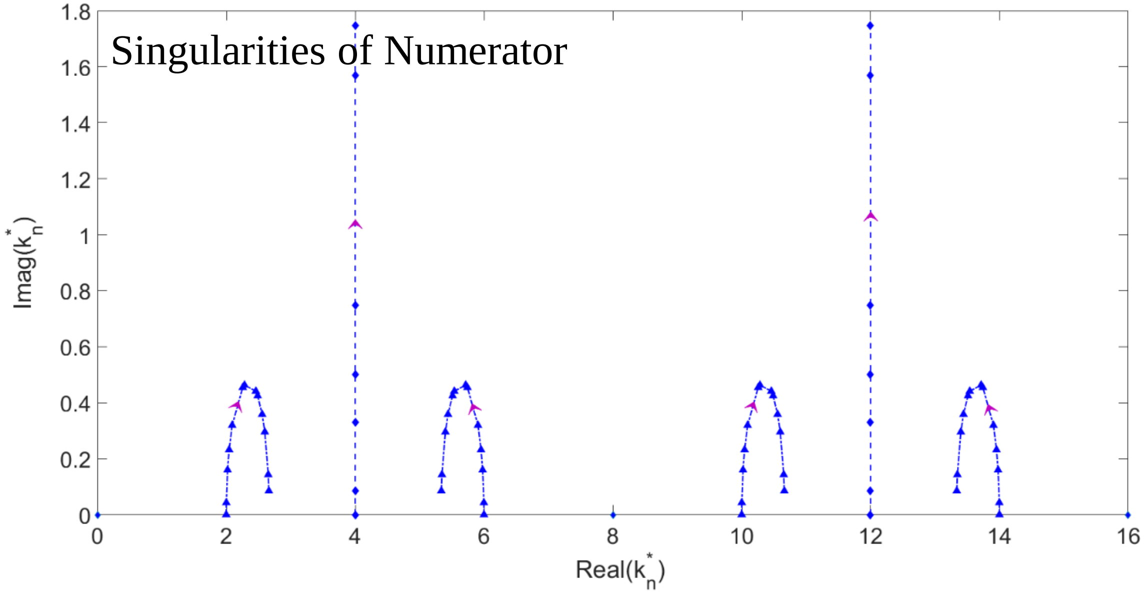


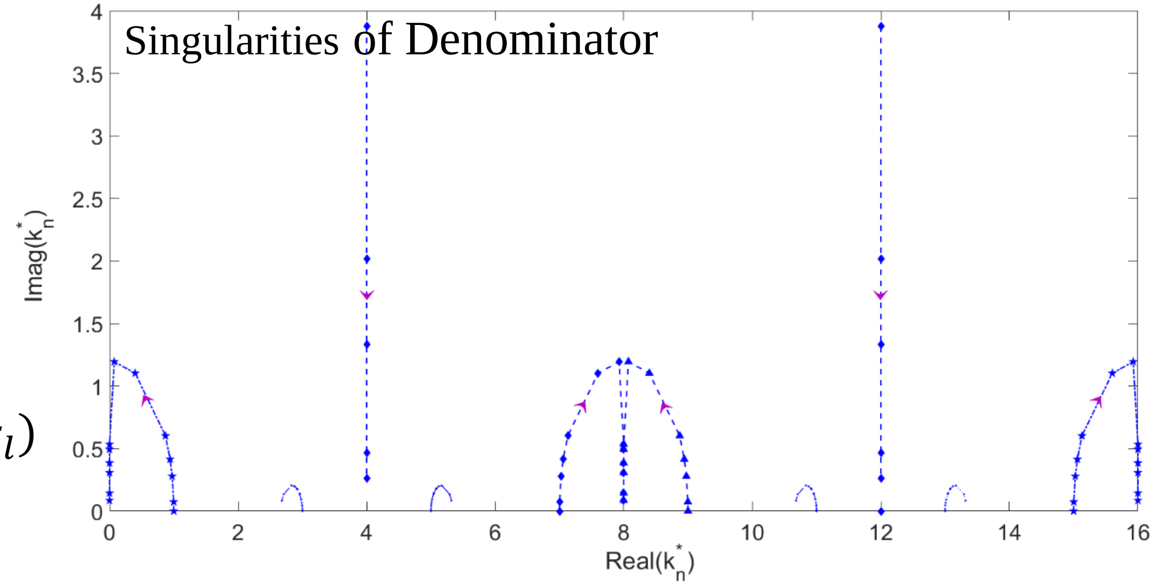


FRF : $h^D(k) = \dfrac{-\frac{ia}{gA}\sin kl + \frac{a}{gA}\frac{Z_C}{Z_L}\sin k(l-x_l)\sin kx_l}{\cos kl + i\frac{Z_C}{Z_L}\cos k(l-x_l)\sin kx_l}$

Points extracting from the two figures:

1. Leak size ↑ , Real(k_n^*) ↑ ;
2. Leak size ↑ to certain amount, some of the neighboring singularities shift to combine to one singularity;
3. Leak size ↑ to certain amount, the resonance wavenumber for pipe section $(l - x_l)$ come into effects.
4. Leak size ↑ to be very large leak, the pipe system (l) will resonance at the resonance wavenumber for pipe section $(l - x_l)$





Summary:

1. When the leak size is small $0 < \frac{z_C}{z_L} < 1.0$, the leak-induced shift is also very small that could be neglected in some leak detection method, which corresponds to Lee et al 2005;

2. When the leak size becomes very large, FRF will be greatly changed, the original resonance frequencies will disappeared, while additional resonance frequencies will come into effects, which corresponds to the experitmental finding from Ferrante et al 2001, and partly corresponds to the finding by Mpesha et al. 2001.

5 Conclusion

Conclusions:

1. In this study, the resonance condition for leaking pipe is studied, the leak-induced resonance frequency shift is analytically and numerically proved to be exist, and is also proved to be complex number.

2. In small leak cases, first order Taylor expansion is accurately enough for capturing the leak induced shift;

3. In large leak cases, some of the resonance wavenumber dispeared, while additional resonance wavenumber comes into effect

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