

Fig. 1 Sketches of irregular blockages in practical water pipelines

blockage radius changes continuously along the axial direction



Fig. 2 Simplified regular blockage in previous studies

blockage radius keeps constant along the axial direction

Transient wave-irregular blockage interaction in pipelines

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Background - experimental system & settings

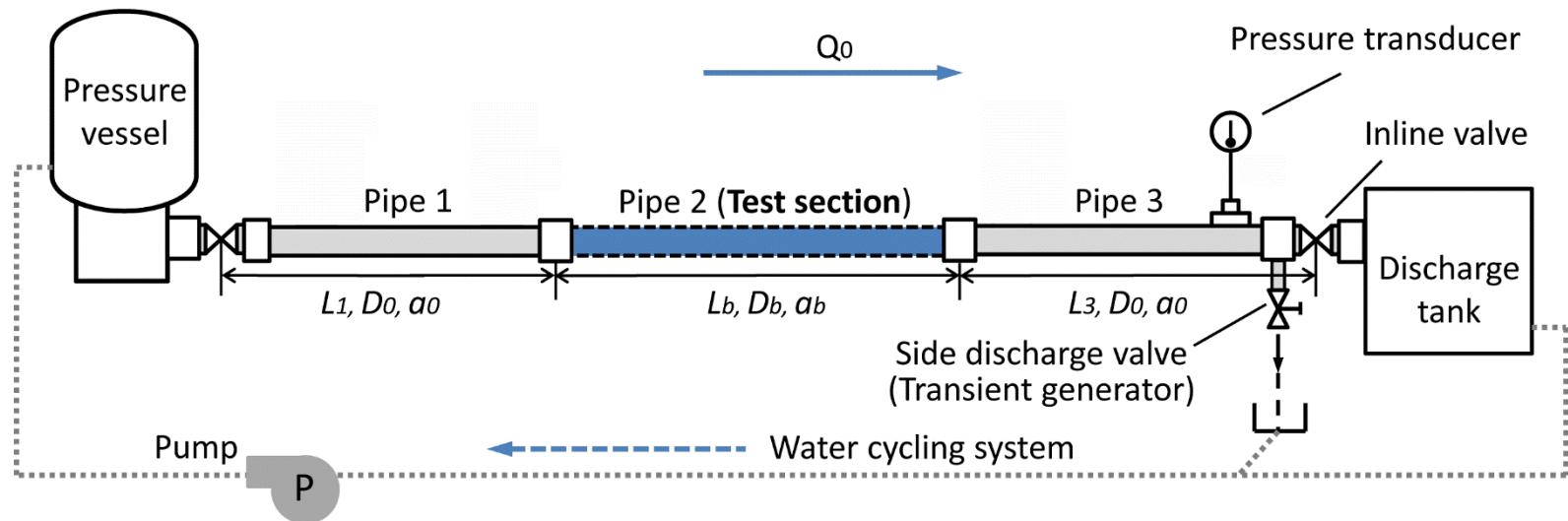


Fig. 1 Sketch of experimental test system

The regular blockage is severer than the irregular blockage.

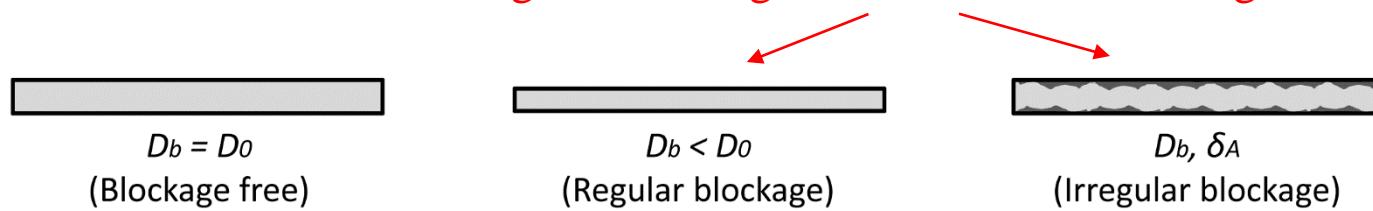
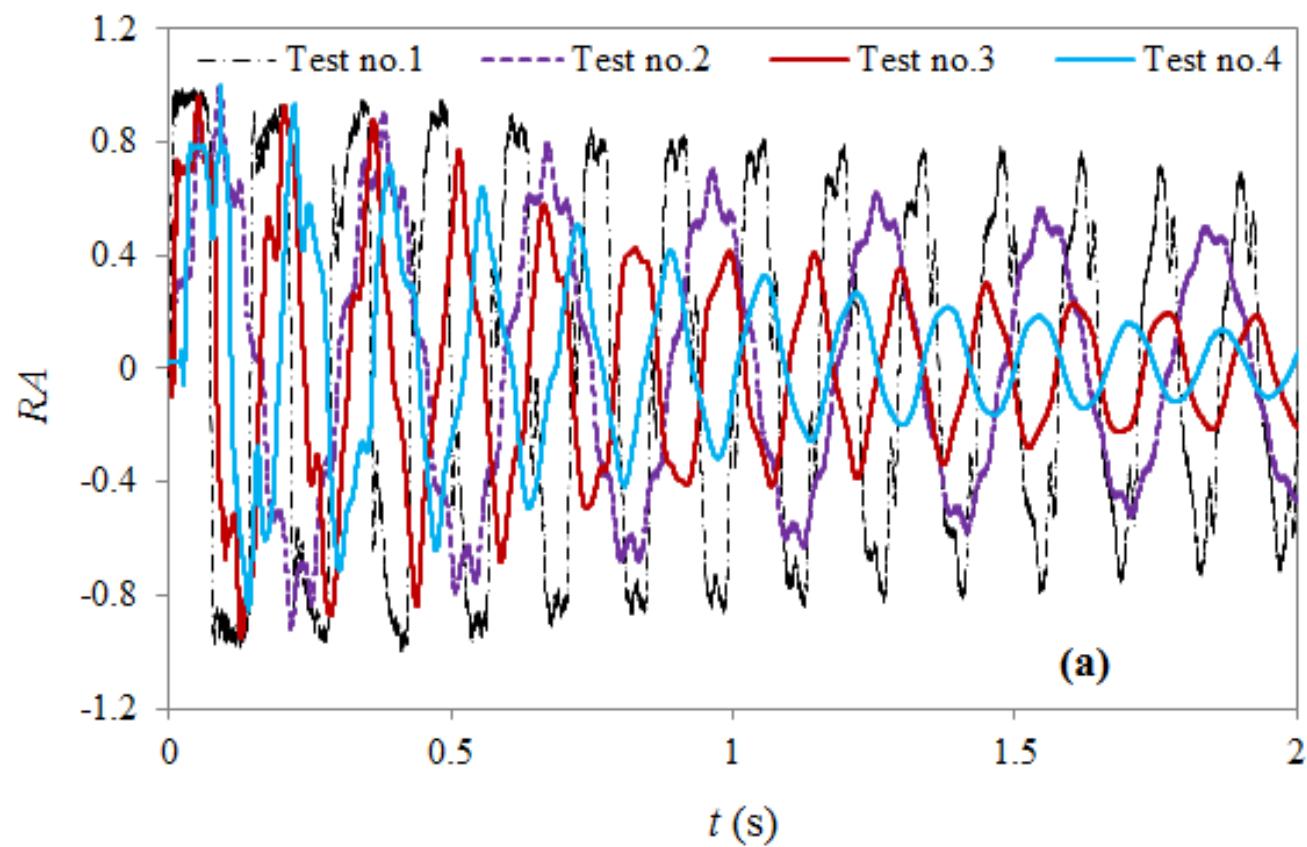


Fig. 2 Inserted sections for different tests

Background - experimental tests & observations

Table

Test no. and blockage type
1: Blockage-free
2: Regular blockage
3: Irregular aggregate blockage
4: Irregular coir blockage



Significant influence of blockage irregularities!!!

Background - TFR-based blockage detection

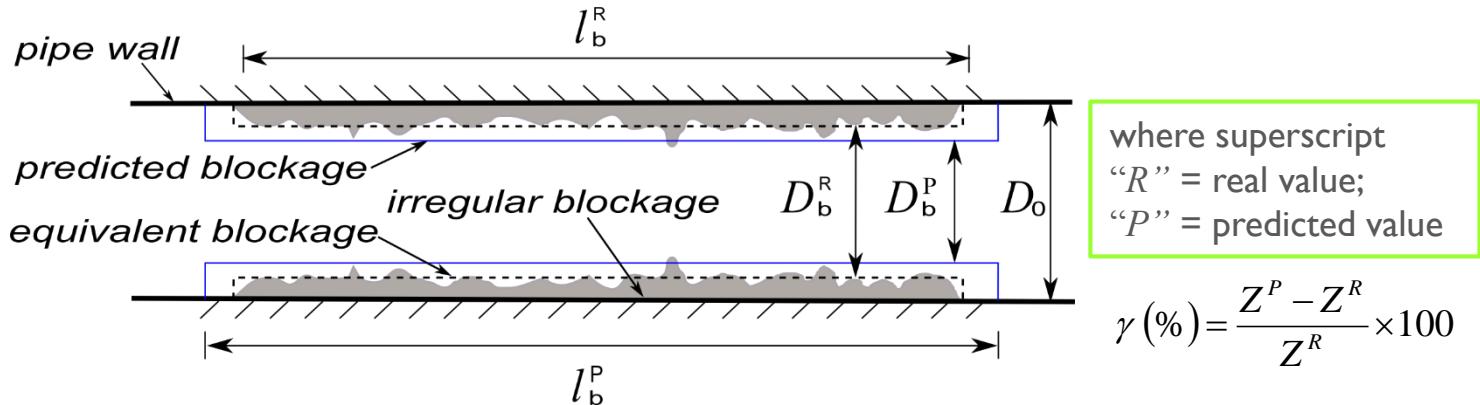


Fig. 1 Schematic of irregular blockage detection in the pipeline

Table 1 Detection results and accuracy of transient-based blockage detection

Test no. and test case	Blockage size (mm)		Blockage length (m)		Blockage location (m)			
2: Regular blockage	Inaccuracy / Failure by current TFR-based method!!!							
3: Rough aggregate blockage	59.3	52.08	-12.18	5.59	17.89	220	20.41	21.57
4: Rough coir blockage	59.6	52.73	-11.54	5.54	14.67	165	20.41	21.38

Background - research scope

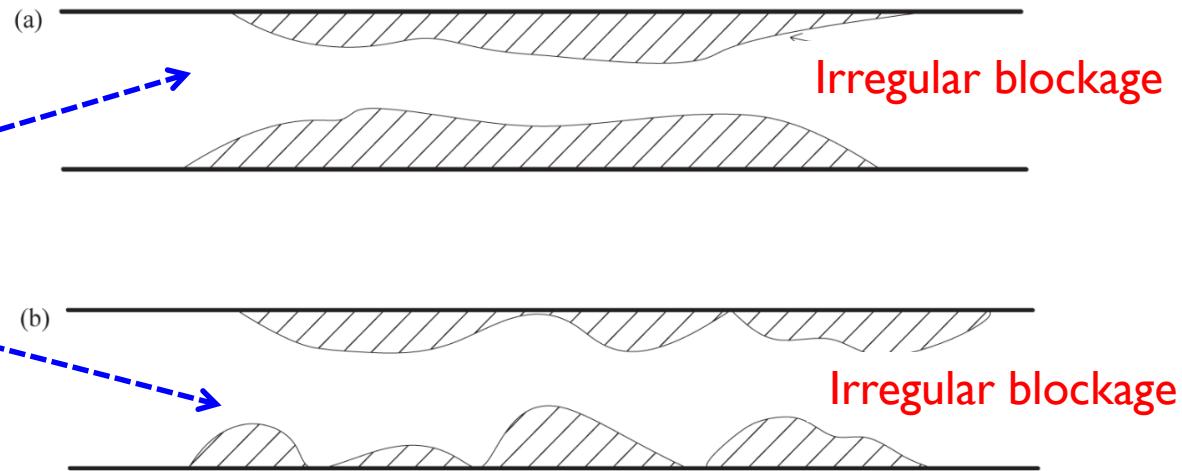


Fig. 1 Sketches of irregular blockages in practical water pipelines

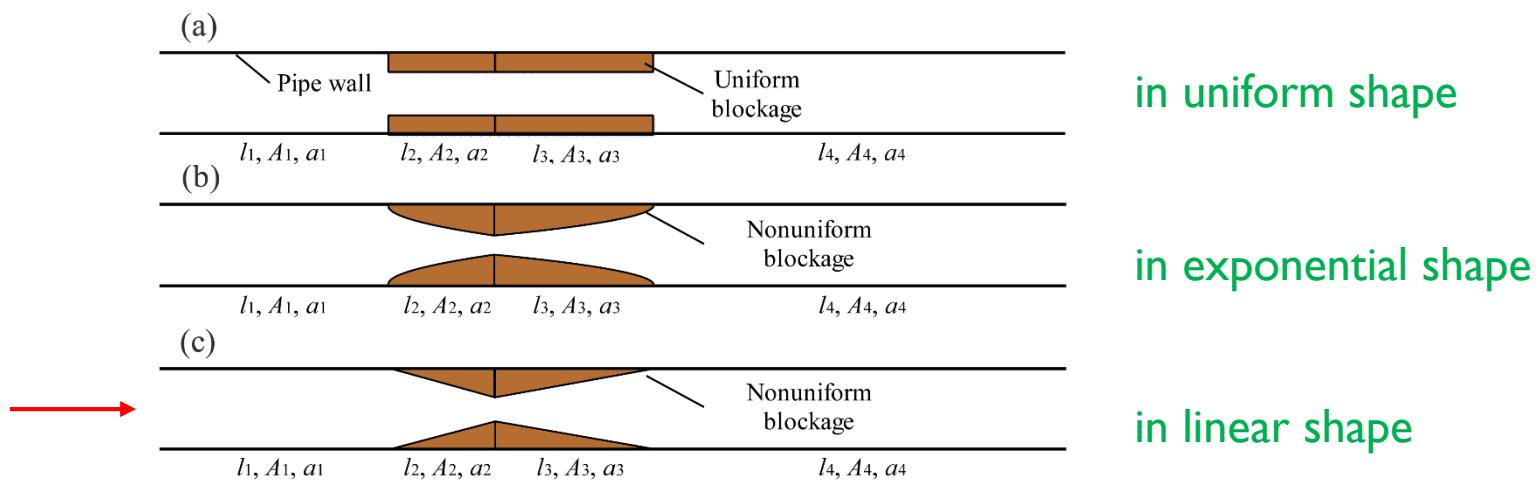


Fig. 2 Blockages in various shapes

Transient wave behaviors - wave equation

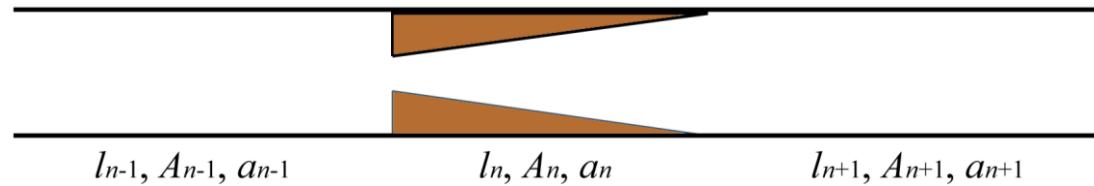


Fig. 1 Linear blockage used in analytical investigation

Water hammer

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho U A)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial(\rho U A)}{\partial t} + A \frac{\partial P}{\partial x} = 0 \quad (2)$$

Equation of state

$$\frac{1}{a^2} = \frac{1}{A} \frac{d(\rho A)}{dP}$$

Key assumptions

1. Frictionless
2. Elastic pipe wall



$$A \frac{\partial^2 P}{\partial t^2} = a^2 \frac{\partial}{\partial x} \left(A \frac{\partial P}{\partial x} \right)$$



Rewrite

$$\frac{\partial^2 P}{\partial t^2} - a^2 \frac{\partial^2 P}{\partial x^2} = a^2 \frac{A'}{A} \frac{\partial P}{\partial x} \quad \text{Wave equation}$$

Transient wave behaviors - wave equation

Wave equation for a conduit with varying cross-sectional area

$$\frac{\partial^2 P}{\partial t^2} - a^2 \frac{\partial^2 P}{\partial x^2} = a^2 \frac{A'}{A} \frac{\partial P}{\partial x}$$



$$P = P_0 + p^*$$

where P_0 = mean pressure; p^* = pressure deviation from the mean

$$\frac{\partial^2 p^*}{\partial t^2} - a^2 \frac{\partial^2 p^*}{\partial x^2} = a^2 \frac{A'}{A} \frac{\partial p^*}{\partial x}$$



$$p^*(x, t) = \operatorname{Re}(p(x)e^{i\omega t})$$

where ω = angular frequency; p = complex variable

$$\frac{d^2 p}{dx^2} + \frac{A'}{A} \frac{dp}{dx} + k^2 p = 0$$



for Section n

$$\frac{d^2 p_n}{dx^2} + \frac{A'_n}{A_n} \frac{dp_n}{dx} + k_0^2 p_n = 0$$

This is called **Webster's Horn Equation** in acoustics

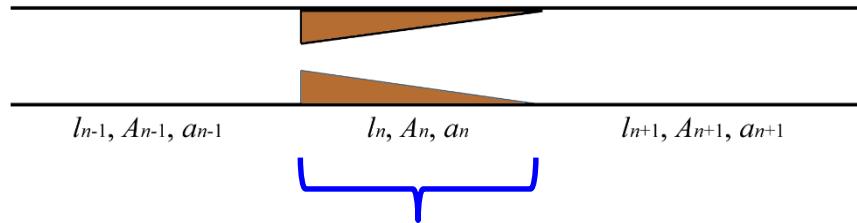


Equation for n -section

Transient wave behaviors - plane wave solutions

Wave equation for pipe Section n

$$\frac{d^2 p_n}{dx^2} + \frac{A'_n}{A_n} \frac{dp_n}{dx} + k_0^2 p_n = 0$$



$$\frac{d^2 p_n}{dx^2} + \frac{2s}{sx + R_{\min}} \frac{dp_n}{dx} + k_0^2 p_n = 0$$

$$p_n = \frac{e^{\alpha x}}{sx + R_{\min}}$$

$$\alpha^2 + k_0^2 = 0 \quad \text{Characteristic equation}$$

Solutions for α

$$\alpha_1 = ik_0 \quad \alpha_2 = -ik_0$$

Solutions for p_n

$$(p_n)_1 = \frac{e^{ik_0 x}}{sx + R_{\min}} \quad (p_n)_2 = \frac{e^{-ik_0 x}}{sx + R_{\min}}$$

Two special solutions for p_n

Pipe radius $r_n(x) = sx + R_{\min}$

Cross area $A_n(x) = \pi r_n^2 = \pi(sx + R_{\min})^2$

Derivative $A'_n = 2s\pi(sx + R_{\min})$

where s = slope; R_{\min} = pipe radius at L.B.

where α = a constant remains to be determined

$$\xrightarrow{\text{Superposition}} p_n = \frac{I e^{ik_0 x} + R e^{-ik_0 x}}{sx + R_{\min}}$$

where
 I and R = constants

General solutions

Transient wave behaviors - visualization

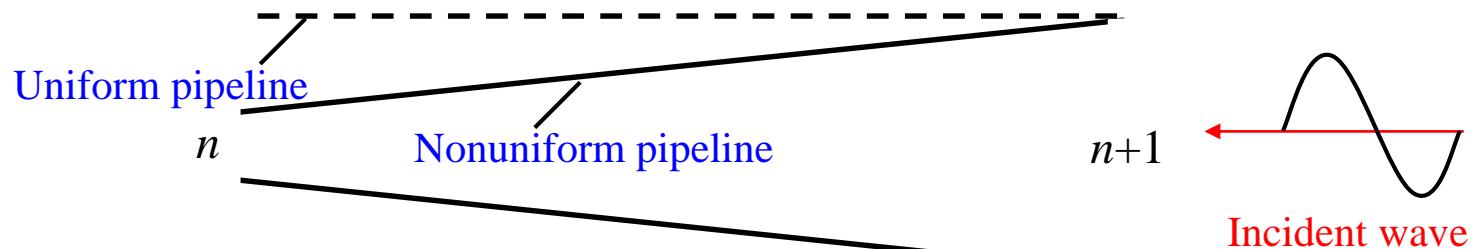


Fig. 1 Uniform pipeline & nonuniform pipelines

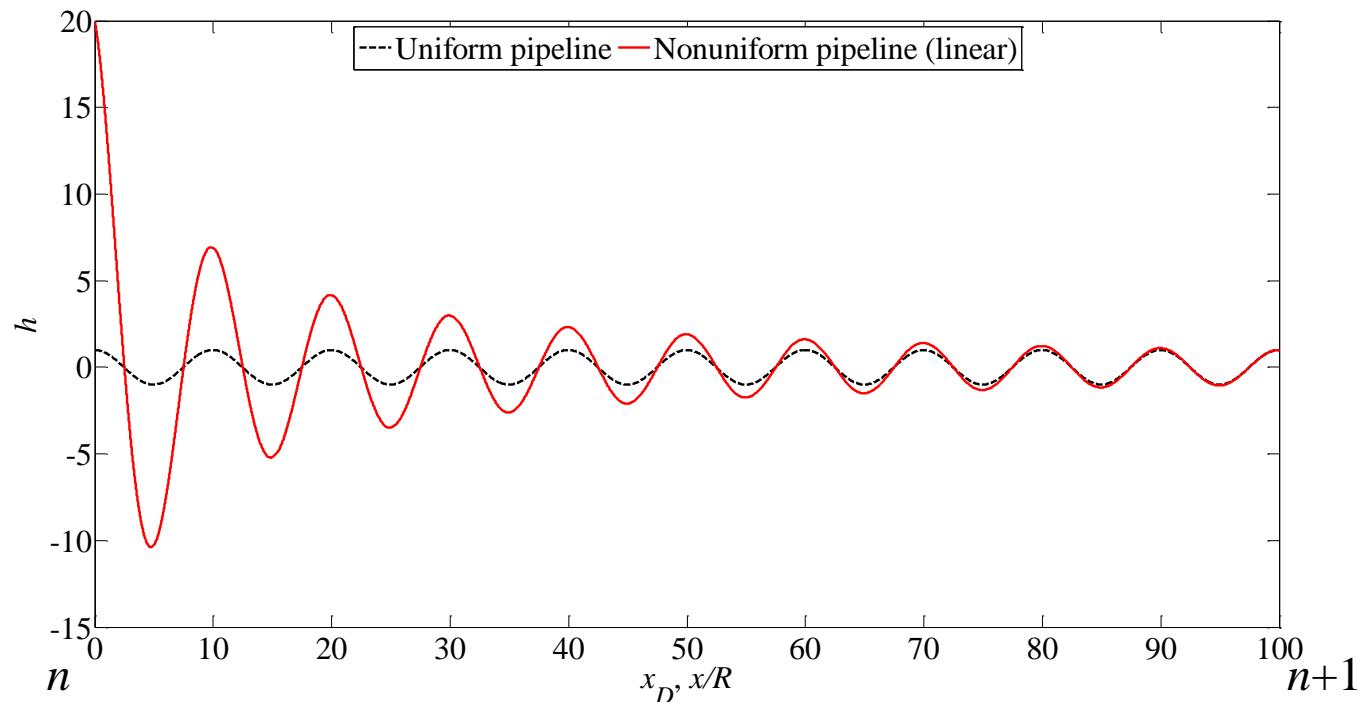
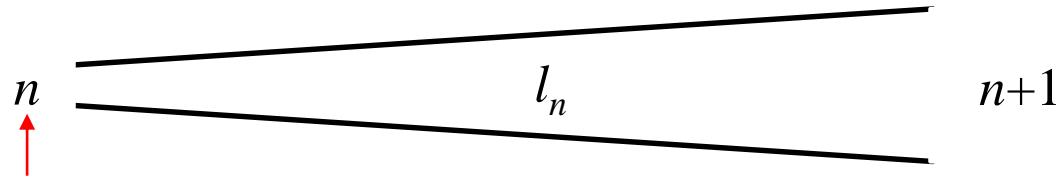


Fig. 2 Wave evolution as it propagates in the pipeline

Transfer matrix - derivation (follow Chaudhry 2014)

$$p = \frac{e^{ik_0x}I + e^{-ik_0x}R}{sx + R_{\min}}$$

$$q = -\frac{A}{i\omega\rho_0} \frac{[ik_0(sx + R_{\min}) - s]e^{ik_0x}I - [ik_0(sx + R_{\min}) + s]e^{-ik_0x}R}{(sx + R_{\min})^2}$$



At point n ($x = 0$)

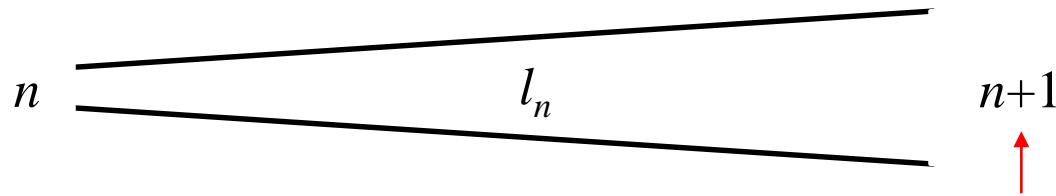
$$p_n = \frac{I + R}{R_{\min}}$$

$$q_n = -\frac{A_n}{i\omega\rho_0} \frac{(ik_0R_{\min} - s)I - (ik_0R_{\min} + s)R}{R_{\min}^2}$$



$$I = \frac{(ik_0R_{\min} + s)p_n - \frac{i\omega\rho_0}{A_n}R_{\min}q_n}{2ik_0} \quad R = \frac{(ik_0R_{\min} - s)p_n + \frac{i\omega\rho_0}{A_n}R_{\min}q_n}{2ik_0}$$

Transfer matrix - derivation (follow Chaudhry 2014)



At point $n+1$ ($x = l_n$)

$$q_{n+1} = U_{11}q_n + U_{12}p_n$$

$$p_{n+1} = U_{21}q_n + U_{22}p_n$$

$$\begin{pmatrix} q \\ p \end{pmatrix}_{n+1} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}_n$$

Transfer matrix for a linear pipeline

$$U_{11} = -\frac{iR_{\min}}{2k_0(sl_n + R_{\min})^2} \frac{A_{n+1}}{A_n} \left\{ [ik_0(sl_n + R_{\min}) - s]e^{ik_0l_n} + [ik_0(sl_n + R_{\min}) + s]e^{-ik_0l_n} \right\}$$

$$U_{12} = \frac{A_{n+1}}{2\omega\rho_0 k_0 (sl_n + R_{\min})^2} \left\{ [ik_0(sl_n + R_{\min}) - s](ik_0R_{\min} + s)e^{ik_0l_n} - [ik_0(sl_n + R_{\min}) + s](ik_0R_{\min} - s)e^{-ik_0l_n} \right\}$$

$$U_{21} = \frac{\omega\rho_0 R_{\min}}{2k_0(sl_n + R_{\min})A_n} (e^{-ik_0l_n} - e^{ik_0l_n})$$

$$U_{22} = \frac{1}{2ik_0(sl_n + R_{\min})} [(ik_0R_{\min} + s)e^{ik_0l_n} + (ik_0R_{\min} - s)e^{-ik_0l_n}]$$

Let $U_{11} = 0 \longrightarrow 2i[k_0(sl_n + R_{\min})\cos(k_0l_n) - s\sin(k_0l_n)] = 0$ Resonance frequency

Transfer matrix - visualization

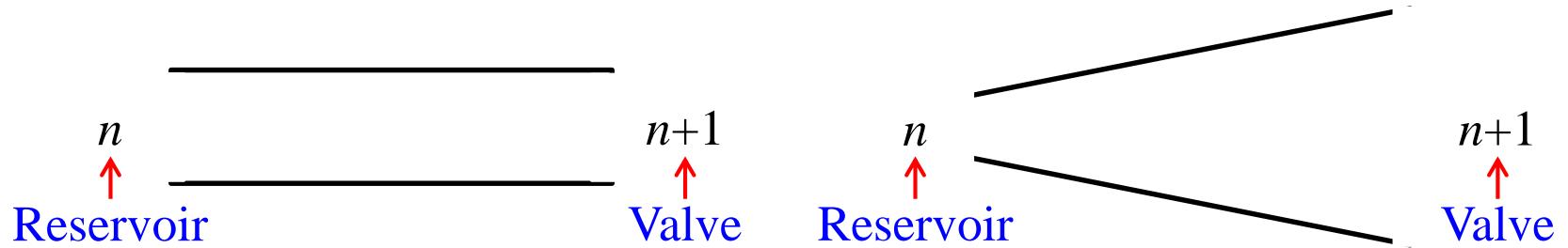


Fig. 1 Sketch of a reservoir-pipe-valve system

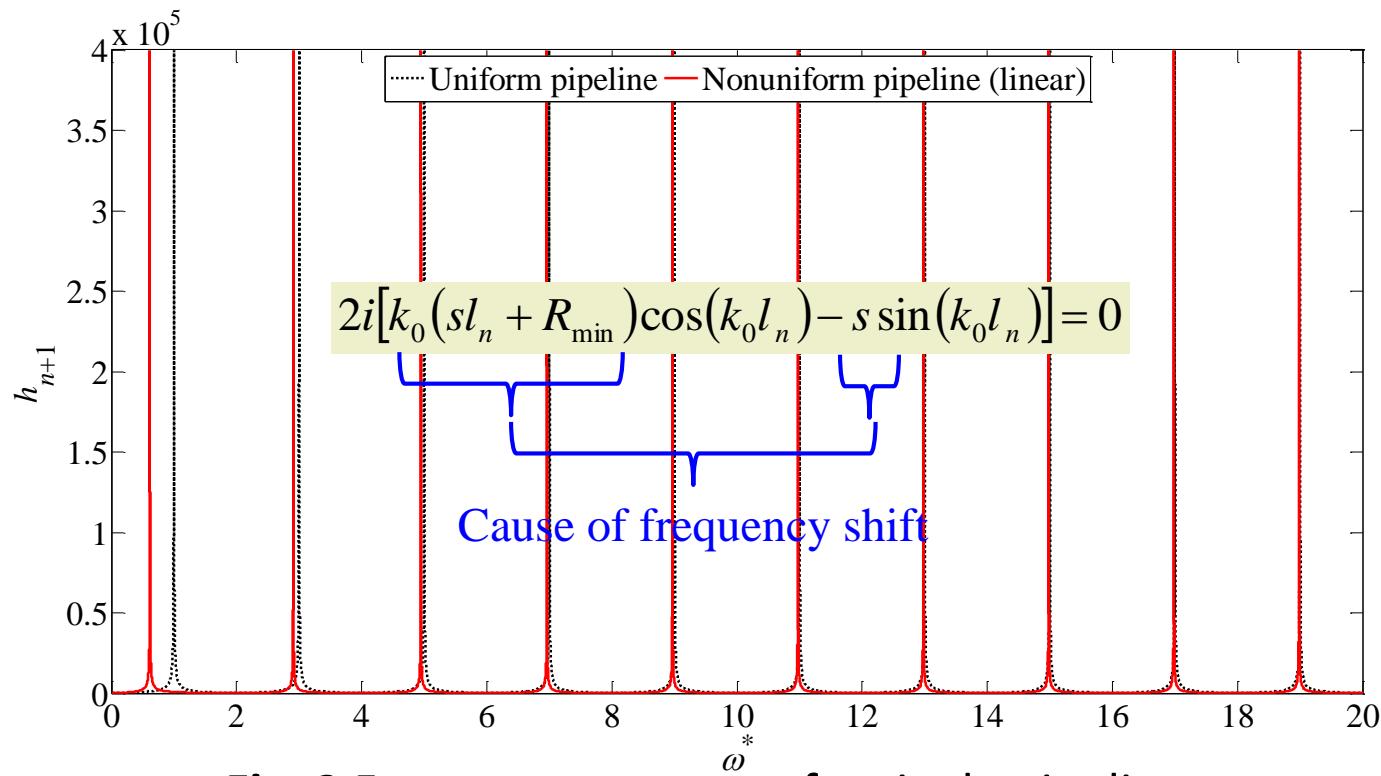
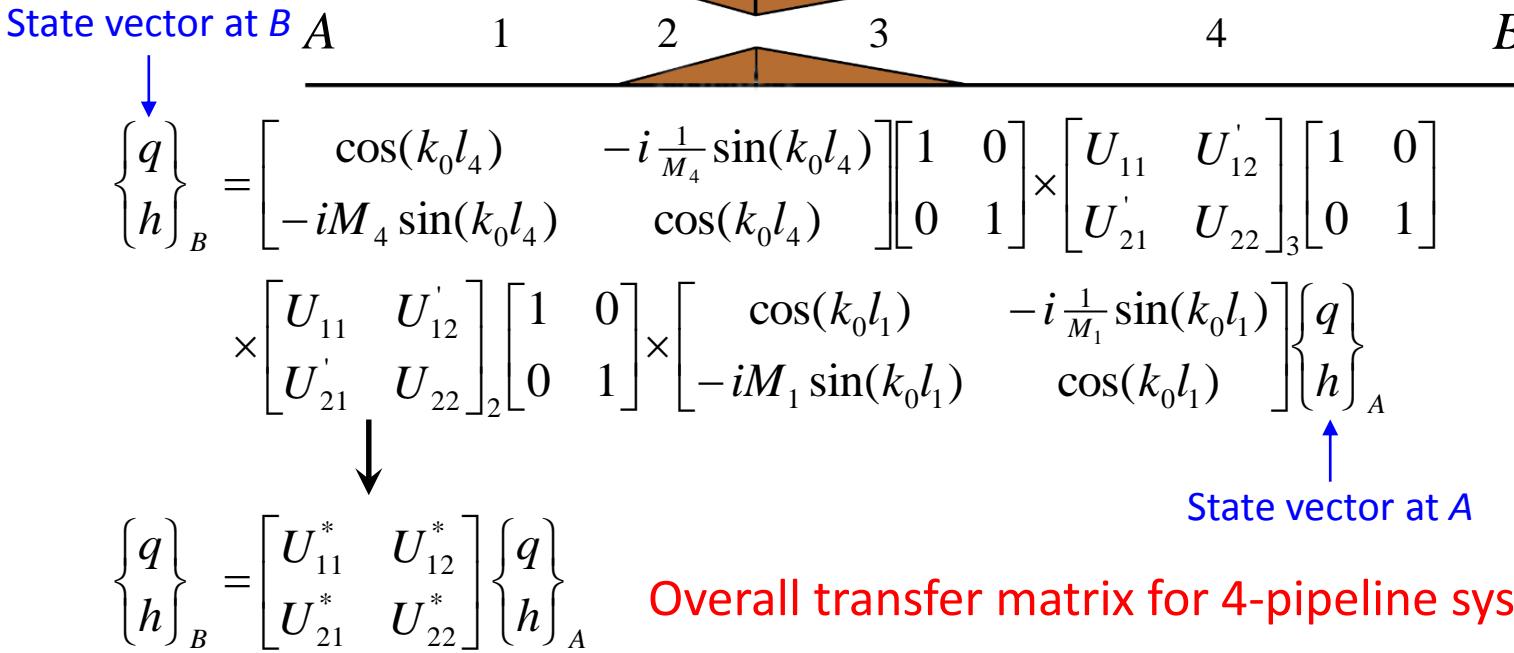


Fig. 2 Frequency response for single pipeline

Numerical applications - 4 pipelines



Overall transfer matrix for 4-pipeline system

$$U_{11}^* = \left\{ \left[\cos(k_0 l_4) (U_{11})_3 - i(1/M_4) \sin(k_0 l_4) (U_{21}')_3 \right] (U_{11})_2 + \left[\cos(k_0 l_4) (U_{12}')_3 - i(1/M_4) \sin(k_0 l_4) (U_{22})_3 \right] (U_{21}')_2 \right\} \cos(k_0 l_1)$$

$$+ \left\{ \left[\cos(k_0 l_4) (U_{11})_3 - i(1/M_4) \sin(k_0 l_4) (U_{21}')_3 \right] (U_{12}')_2 + \left[\cos(k_0 l_4) (U_{12}')_3 - i(1/M_4) \sin(k_0 l_4) (U_{22})_3 \right] (U_{22})_2 \right\} [-i M_1 \sin(k_0 l_1)]$$

$$U_{12}^* = \left\{ \left[\cos(k_0 l_4) (U_{11})_3 - i(1/M_4) \sin(k_0 l_4) (U_{21}')_3 \right] (U_{11})_2 + \left[\cos(k_0 l_4) (U_{12}')_3 - i(1/M_4) \sin(k_0 l_4) (U_{22})_3 \right] (U_{21}')_2 \right\} [-i(1/M_1) \sin(k_0 l_1)]$$

$$+ \left\{ \left[\cos(k_0 l_4) (U_{11})_3 - i(1/M_4) \sin(k_0 l_4) (U_{21}')_3 \right] (U_{12}')_2 + \left[\cos(k_0 l_4) (U_{12}')_3 - i(1/M_4) \sin(k_0 l_4) (U_{22})_3 \right] (U_{22})_2 \right\} \cos(k_0 l_1)$$

$$U_{21}^* = \left\{ \left[-i M_4 \sin(k_0 l_4) (U_{11})_3 + \cos(k_0 l_4) (U_{21}')_3 \right] (U_{11})_2 + \left[-i M_4 \sin(k_0 l_4) (U_{12}')_3 + \cos(k_0 l_4) (U_{22})_3 \right] (U_{21}')_2 \right\} \cos(k_0 l_1)$$

$$+ \left\{ \left[-i M_4 \sin(k_0 l_4) (U_{11})_3 + \cos(k_0 l_4) (U_{21}')_3 \right] (U_{12}')_2 + \left[-i M_4 \sin(k_0 l_4) (U_{12}')_3 + \cos(k_0 l_4) (U_{22})_3 \right] (U_{22})_2 \right\} [-i M_1 \sin(k_0 l_1)]$$

$$U_{22}^* = \left\{ \left[-i M_4 \sin(k_0 l_4) (U_{11})_3 + \cos(k_0 l_4) (U_{21}')_3 \right] (U_{11})_2 + \left[-i M_4 \sin(k_0 l_4) (U_{12}')_3 + \cos(k_0 l_4) (U_{22})_3 \right] (U_{21}')_2 \right\} [-i(1/M_1) \sin(k_0 l_1)]$$

$$+ \left\{ \left[-i M_4 \sin(k_0 l_4) (U_{11})_3 + \cos(k_0 l_4) (U_{21}')_3 \right] (U_{12}')_2 + \left[-i M_4 \sin(k_0 l_4) (U_{12}')_3 + \cos(k_0 l_4) (U_{22})_3 \right] (U_{22})_2 \right\} \cos(k_0 l_1)$$

Numerical applications - 4 pipelines

Let $U_{11}^* = 0$ Resonance frequency pattern for 4-pipeline system

$$\frac{1}{4R_0^2 R_{\min} \omega^3} \left\{ \begin{array}{l} -a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 - l_2 - l_3 - l_4)] + a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 + l_2 + l_3 - l_4)] + a^2 R_{\min} s^2 \omega \cos[k_0(l_1 - l_2 - l_3 + l_4)] - \\ 2a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 - l_3 + l_4)] - 2a^2 R_0 s^2 \omega \cos[k_0(l_1 - l_2 + l_3 + l_4)] + 4a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] - \\ a^2 R_{\min} s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] + 4R_0^2 R_{\min} \omega^3 \cos[k_0(l_1 + l_2 + l_3 + l_4)] - a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 - l_4)] - \\ 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 - l_2 - l_3 - l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + \\ a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 - l_4)] - a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - \\ a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 + l_4)] - \\ a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 + l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] - 4aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] \end{array} \right\} = 0$$

(1) If $s = 0$ (blockage-free)

$$\cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0$$

(2) If $s \rightarrow \infty$ ($l_2 = l_3 \sim 0$, discrete blockage)

$$\cos[k_0(l_1 + l_4)] = 0 \quad \sim \quad \cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0$$

Discrete blockage does not induce frequency shift.

(3) If $\omega \rightarrow \infty$ (high frequency)

$$\cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0 \quad \text{Extremely high frequency components do not induce frequency shift.}$$



Numerical applications - numerical validation (MOC)

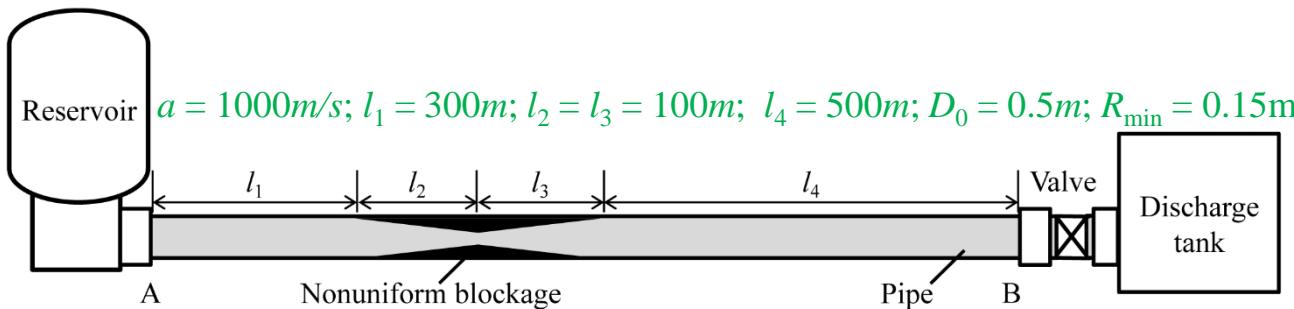


Fig. 1 Sketch of the numerical experiment system (MOC coupled 1D WH Eq.)

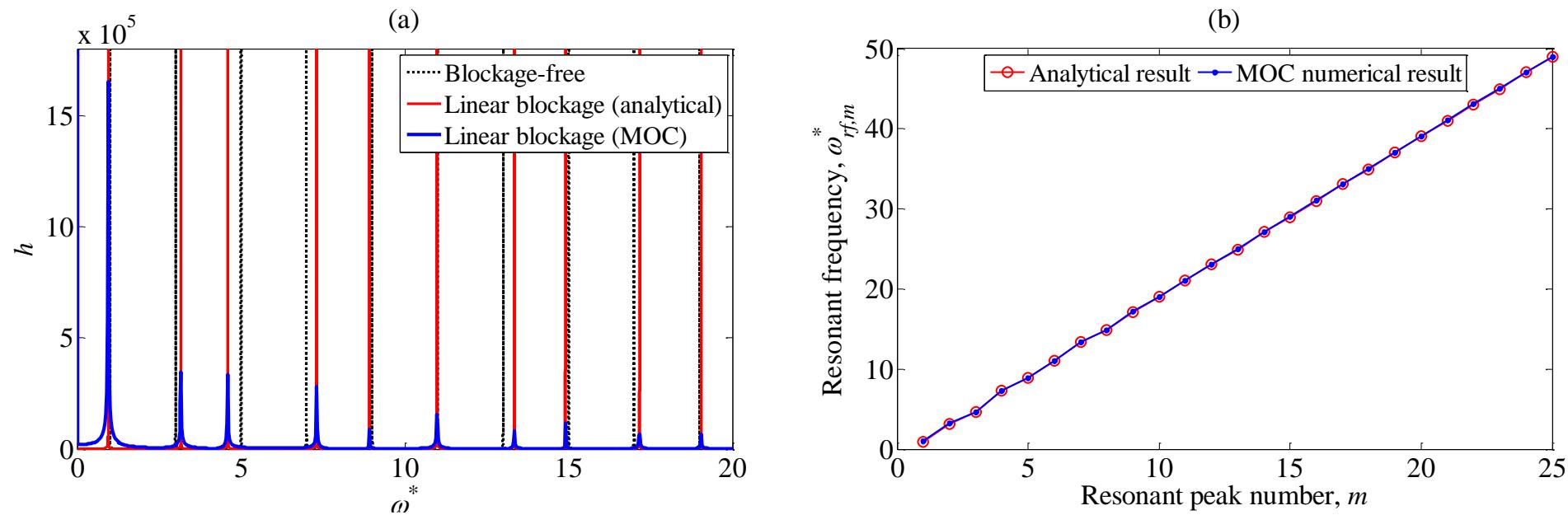
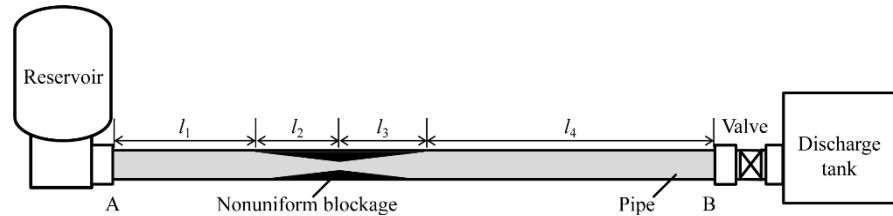
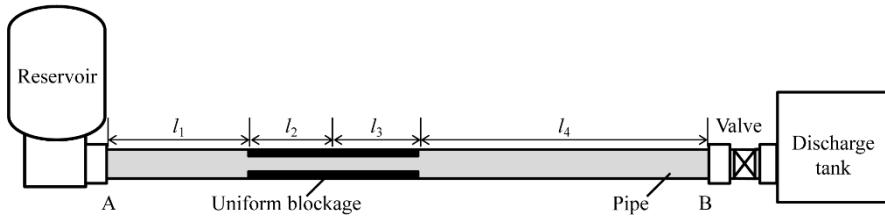
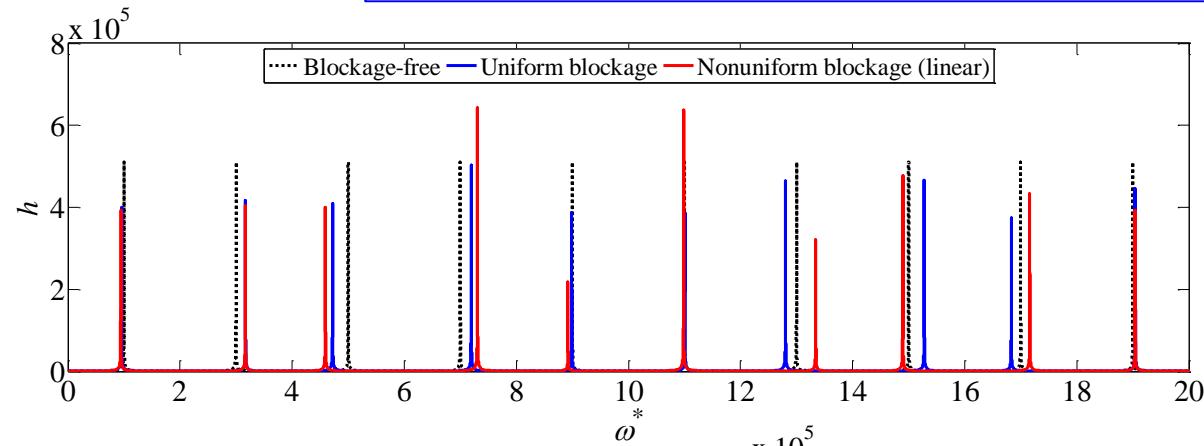


Fig. 2 Validation results for nonuniform blockage (linear)

Numerical applications-uniform & nonuniform blockages

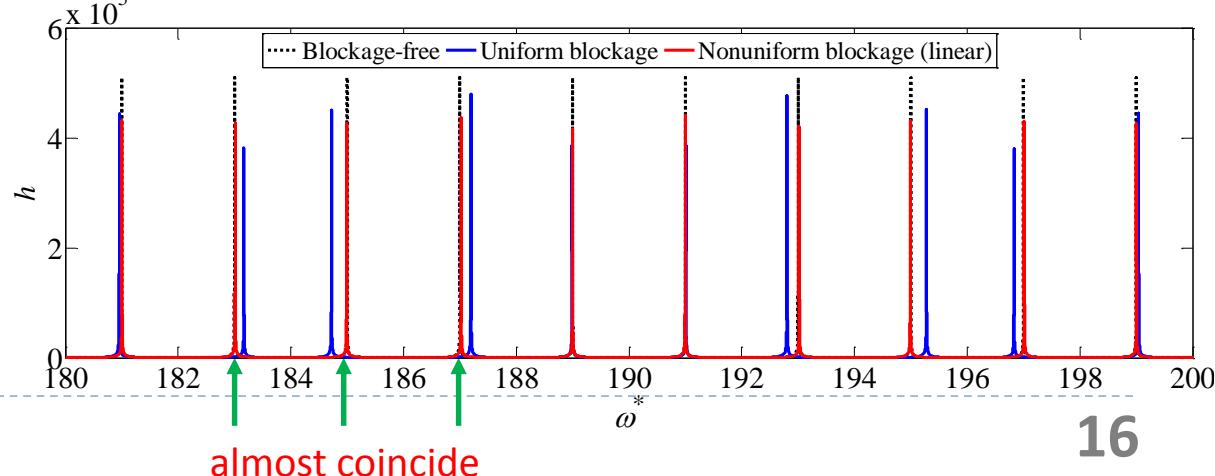


1. Same volume of blockage (uniform & nonuniform)
2. Linearized steady friction

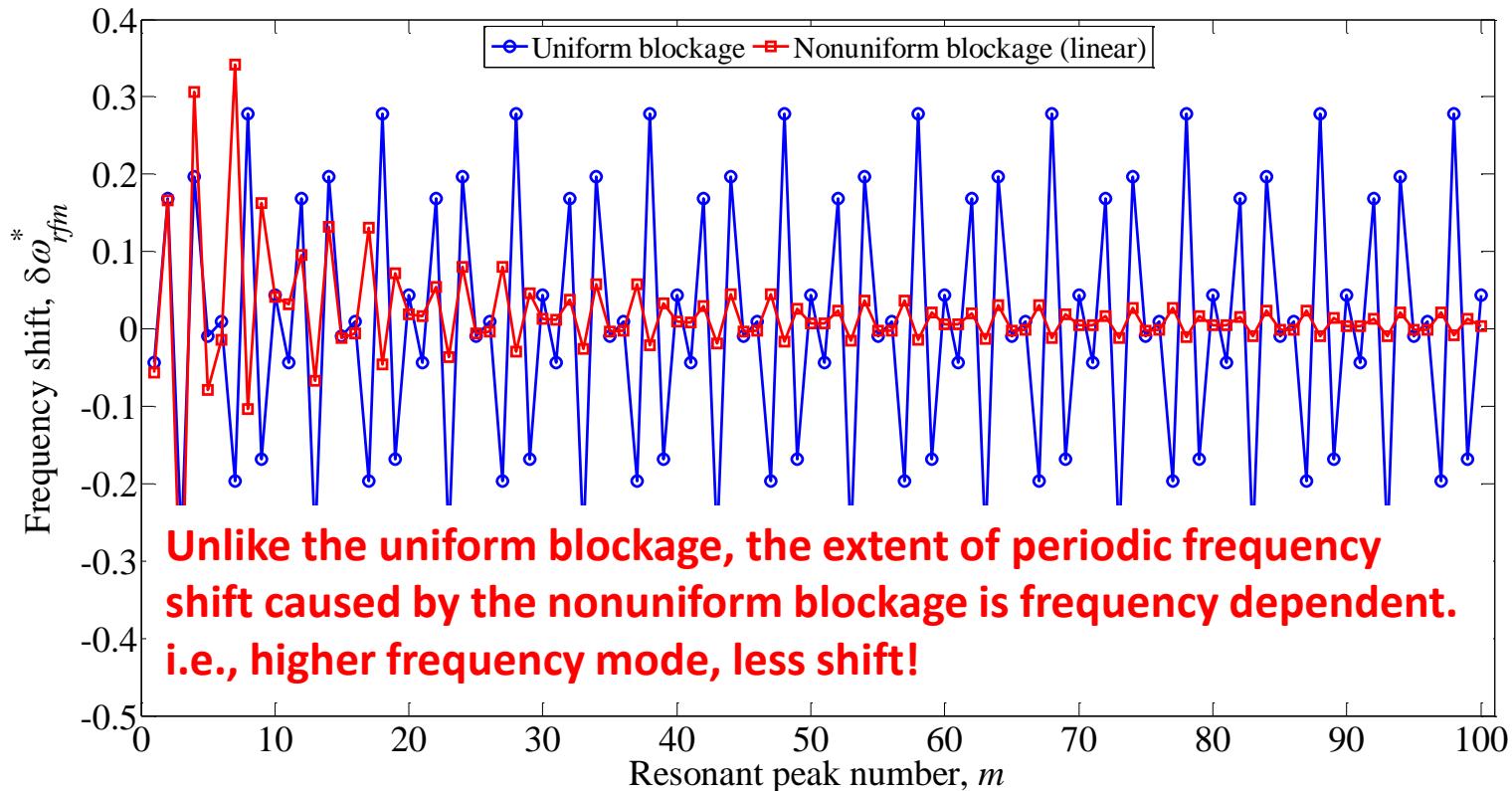
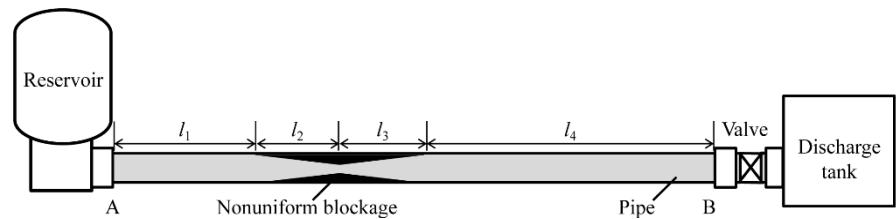
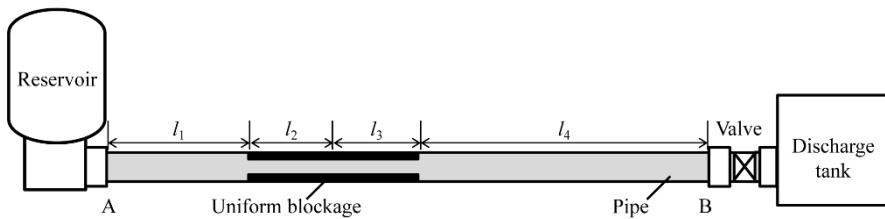


Low frequency components

Relative high frequency components

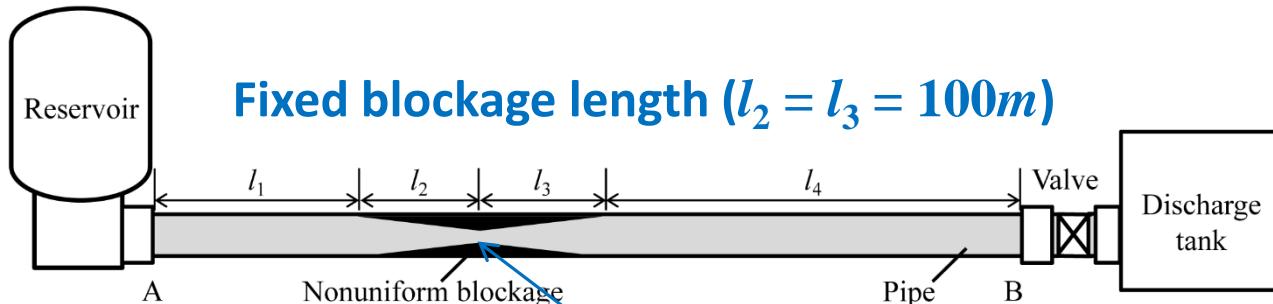


Numerical applications-uniform & nonuniform blockages

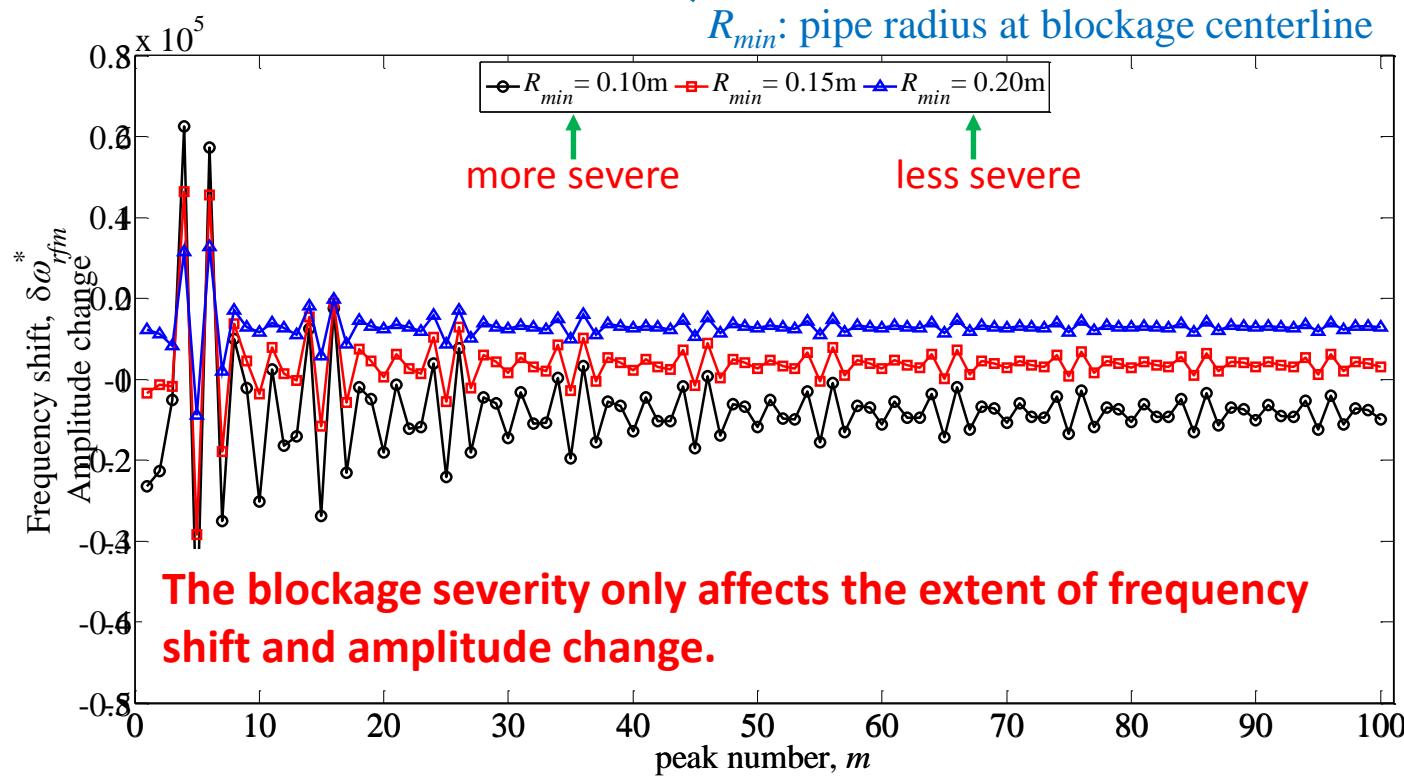


Resonance frequency shift caused by two blockages

Numerical applications - various blockage severity



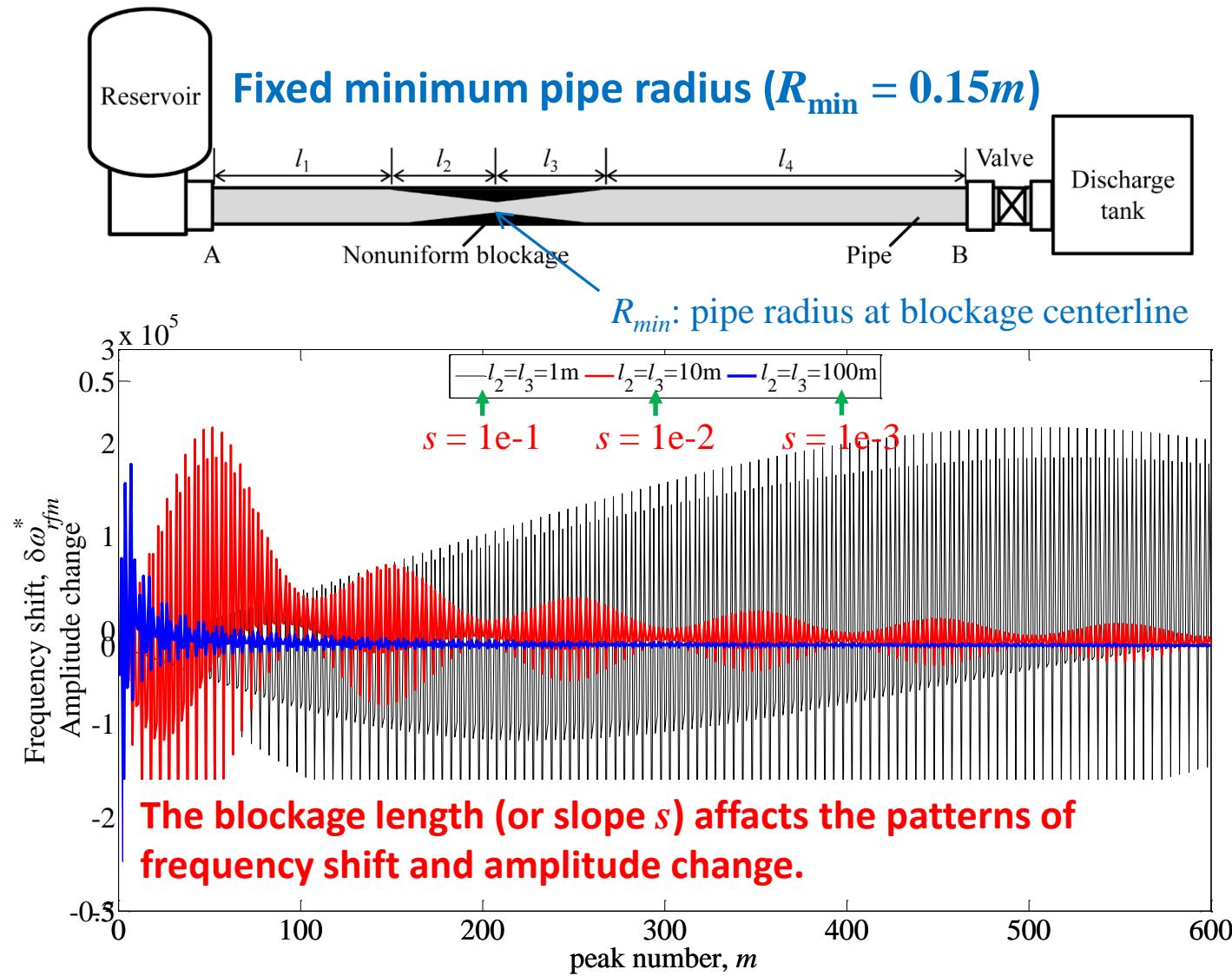
Fixed blockage length ($l_2 = l_3 = 100m$)



Anepliarity shaftage



Numerical applications - various blockage length



AnFptique de chahife



Preliminary Conclusions - (for linear blockage case)

- ▶ Blockage-induced **frequency shift & amplitude change** are frequency-dependent:
 - ▶ Overall, less significant for higher frequency mode; and more significant for larger blockage severity;
 - ▶ The extent of frequency shift / amplitude change periodically decreases with frequency
- ▶ Blockage-induced patterns of **frequency shift & amplitude change (linearized steady friction)**:
 - ▶ Magnitude of shift / change pattern: $\sim s_{blockage}$ (or, $\sim L_0 D_0 / L_b D_b$)
 - ▶ Period of shift / change pattern: $\sim (L/a)_0 / (L/a)_{blockage}$

Summary

1. Transient wave behaviors in a nonuniform blockage;
2. Transfer matrix for a nonuniform blockage is derived and validated by the numerical MOC;
3. Unlike the uniform blockage, the frequency shift of nonuniform blockage becomes less evident for high frequency components;
4. Improved transient-based detection method need to be developed for the nonuniform blockage in future works.

Acknowledgement

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Thank you

June 21st, 2017



Additional slides



Numerical applications - 4 pipelines

Let $U_{11}^* = 0$ Resonance frequency pattern for 4-pipeline system

$$\frac{1}{4R_0^2 R_{\min} \omega^3} \left\{ \begin{array}{l} -a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 - l_2 - l_3 - l_4)] + a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 + l_2 + l_3 - l_4)] + a^2 R_{\min} s^2 \omega \cos[k_0(l_1 - l_2 - l_3 + l_4)] - \\ 2a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 - l_3 + l_4)] - 2a^2 R_0 s^2 \omega \cos[k_0(l_1 - l_2 + l_3 + l_4)] + 4a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] - \\ a^2 R_{\min} s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] + 4R_0^2 R_{\min} \omega^3 \cos[k_0(l_1 + l_2 + l_3 + l_4)] - a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 - l_4)] - \\ 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 - l_2 - l_3 - l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + \\ a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 - l_4)] - a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - \\ a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 + l_4)] - \\ a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 + l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] - 4aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] \end{array} \right\} = 0$$

(1) If $s = 0$ (blockage-free)

$$\cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0$$

(2) If $s \rightarrow \infty$ ($l_2 = l_3 \sim 0$, discrete blockage) SUM(Term(s^3)) = 0 SUM(Term(s^2)) =
 $\cos[k_0(l_1 + l_4)] = 0$ ~ $\cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0$

Discrete blockage does not induce frequency shift.

(3) If $\omega \rightarrow \infty$ (high frequency)

$\cos[k_0(l_1 + l_2 + l_3 + l_4)] = 0$ Extremely high frequency components do not induce frequency shift.



Numerical applications - various blockage length

$$\frac{1}{4R_0^2 R_{\min} \omega^3} \left\{ \begin{aligned} & -a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 - l_2 - l_3 - l_4)] + a^2(2R_0 - R_{\min})s^2 \omega \cos[k_0(l_1 + l_2 + l_3 - l_4)] + a^2 R_{\min} s^2 \omega \cos[k_0(l_1 - l_2 - l_3 + l_4)] - \\ & 2a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 - l_3 + l_4)] - 2a^2 R_0 s^2 \omega \cos[k_0(l_1 - l_2 + l_3 + l_4)] + 4a^2 R_0 s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] - \\ & a^2 R_{\min} s^2 \omega \cos[k_0(l_1 + l_2 + l_3 + l_4)] + \underline{4R_0^2 R_{\min} \omega^3 \cos[k_0(l_1 + l_2 + l_3 + l_4)]} - a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 - l_4)] - \\ & 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 - l_2 - l_3 - l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 - l_3 - l_4)] + \\ & a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 - l_4)] - a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - 2aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 - l_4)] - \\ & a^3 s^3 \sin[k_0(l_1 - l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 + l_2 - l_3 + l_4)] + a^3 s^3 \sin[k_0(l_1 - l_2 + l_3 + l_4)] - \\ & a^3 s^3 \sin[k_0(l_1 + l_2 + l_3 + l_4)] + 4aR_0^2 s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] - 4aR_0 R_{\min} s \omega^2 \sin[k_0(l_1 + l_2 + l_3 + l_4)] \end{aligned} \right\} = 0$$

