



Mode Dispersion Curves

Fluid-Filled Pipe using FEM

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outline



wave quantities

FEM model

mode identification

rigid pipe

deformable pipe

pipe-only



wave quantities

angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ (rad/s)} \quad \text{where} \quad f = \text{frequency (s}^{-1}\text{)}, T = \text{period (s)}$$

spatial frequency

- analogous to the angular frequency, the spatial frequency is defined as:

$$k = \frac{2\pi}{\lambda} \text{ (rad/m)} \quad \text{where} \quad \lambda = \text{wavelength (m)}$$

› wavenumber

phase velocity

- for a given wave and mode, the phase velocity is defined by:

$$c_f = \frac{\lambda}{T} = \frac{\omega}{k} \text{ (m/s)}$$

group velocity

- is associated with the propagation velocity of a group of waves with similar frequency:

$$c_g = \frac{d\omega}{dk} \text{ (m/s)}$$



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automated mode identification

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FEM model

acrylic pipe

geometric properties

- length: 2 m
- diameter: 49.23 mm
- pipe thickness: 4.83 mm

element size

- the minimum element dimension can be calculated using the formula:

$$L_{\max} < \frac{c_w}{n_{\min} \cdot f_{\max}} = \frac{1500}{10 \cdot 40000} = 3.75 \cdot 10^{-3} \text{ m} \quad (n_{\min} \geq 10 \text{ is recommended})$$

element type

- Abaqus' acoustic element, AC3D8 for water
- shell element S4R is used for the pipe

eigensolver → Lanczos algorithm

boundary conditions

material properties

- i. water
 - density: 1000 kg/m³
 - bulk modulus: 2.25 GPa

$$c_w = \sqrt{\frac{2.25 \cdot 10^9}{1000}} = 1500 \text{ (m/s)}$$

$$c_L = \sqrt{\frac{3.138 \cdot 10^9}{1190}} = 1623.87 \text{ (m/s)}$$

- ii. acrylic
 - density: 1190 kg/m³
 - E: 3138.13 GPa
 - Poisson's ratio: 0.39

› number of eigenvalues in
the order of thousands



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mode identification

path definition (1)

description

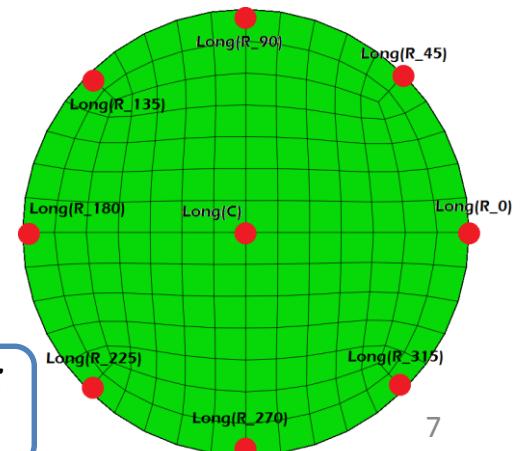
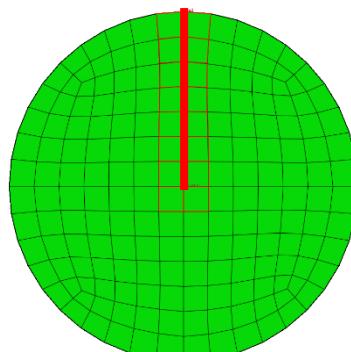
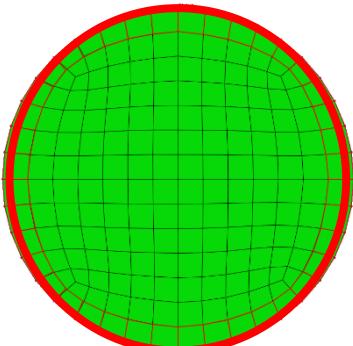
- mode identification is quintessential in plotting the dispersion curves; a mode has a distinct deformation pattern
- “paths” on the system are defined; waveform → mode classification

$(m,n) = (\text{wavelengths in the circumferential path}, \text{zero crossings in the radial path})$

› circumferential

› radial

› longitudinal path



› phase velocity:
 $c_f = \lambda \cdot freq$

outline



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results



rigid pipe

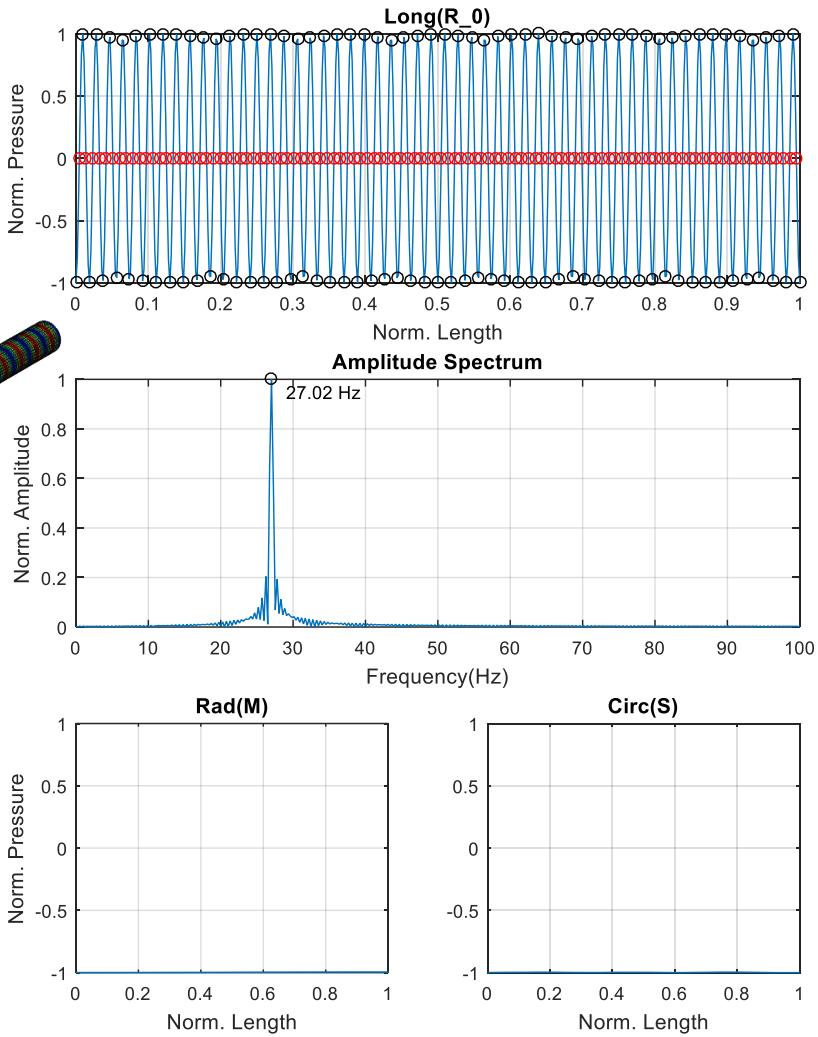
mode (0,0) or planar mode

- the last mode shape in 0-40 kHz
 - 39817.1 Hz
 - 54 wavelengths

| POR |
|------------|
| +1.093e+06 |
| +9.108e+05 |
| +7.286e+05 |
| +5.465e+05 |
| +3.644e+05 |
| +1.822e+05 |
| +8.378e+01 |
| -1.821e+05 |
| -3.642e+05 |
| -5.463e+05 |
| -7.285e+05 |
| -9.106e+05 |
| -1.093e+06 |



› wavelength: $\lambda = 3.7 \cdot 10^{-2} \text{ m}$



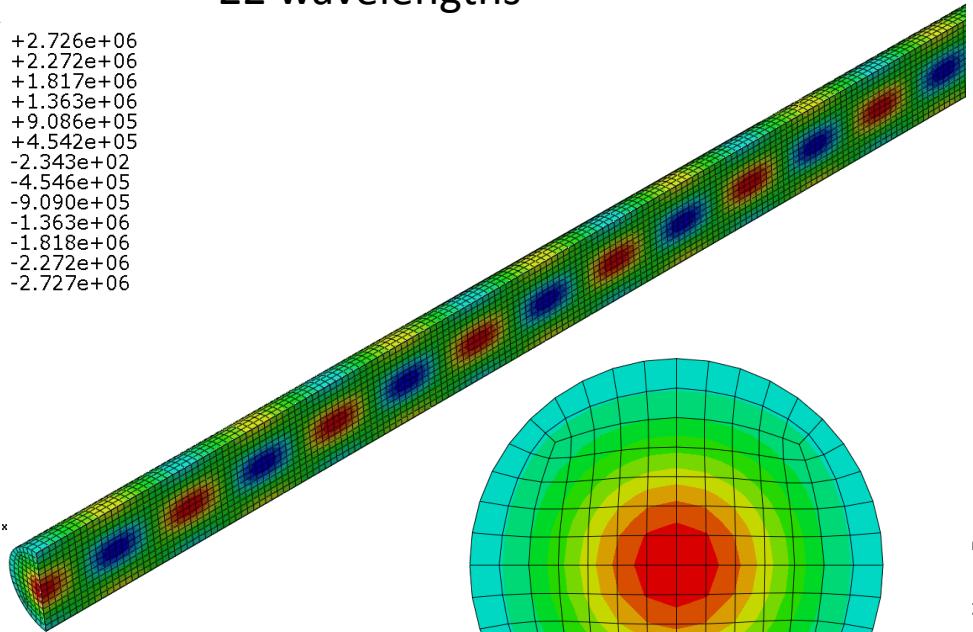
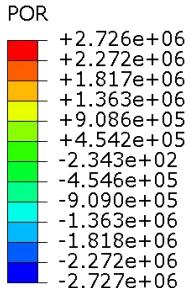
results



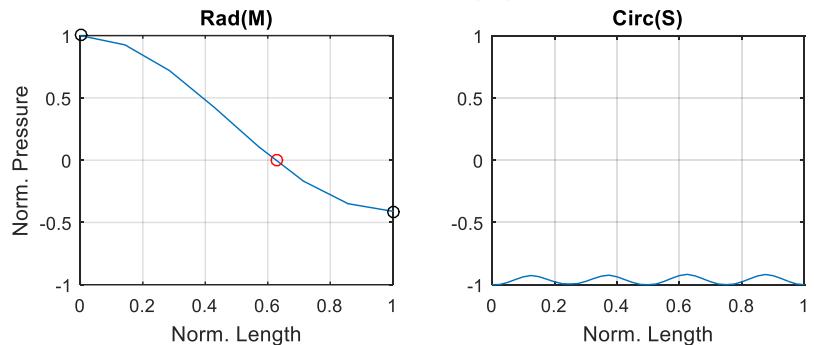
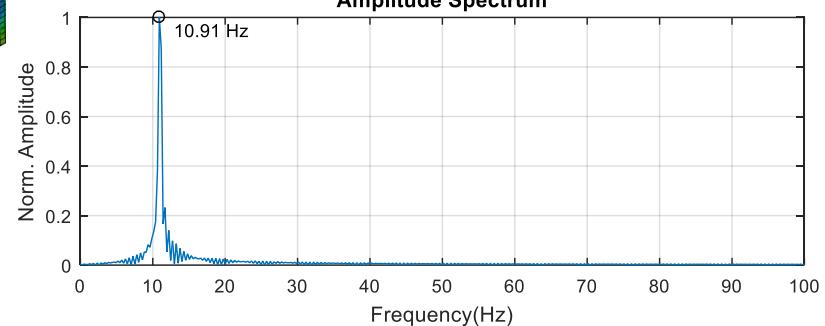
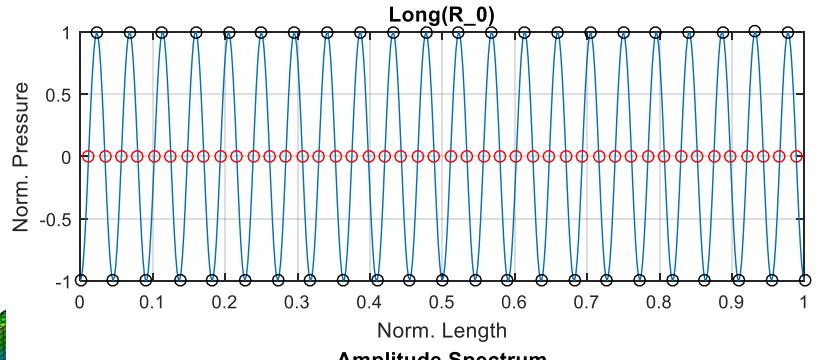
rigid pipe

mode (0,1)

- the last mode shape in 0-40 kHz
 - 39982.2 Hz
 - 22 wavelengths



› wavelength:
 $\lambda = 9.09 \cdot 10^{-2} \text{ m}$



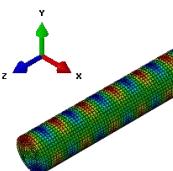
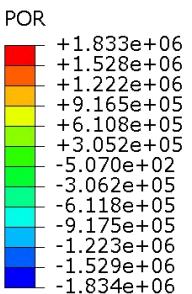


results

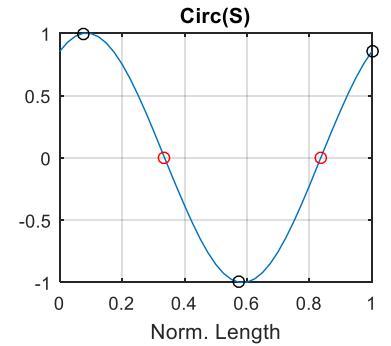
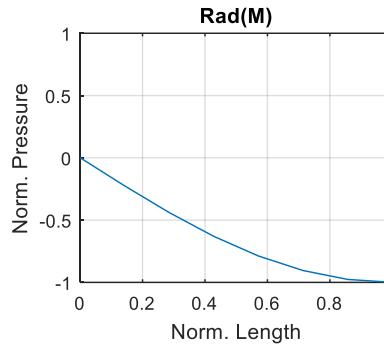
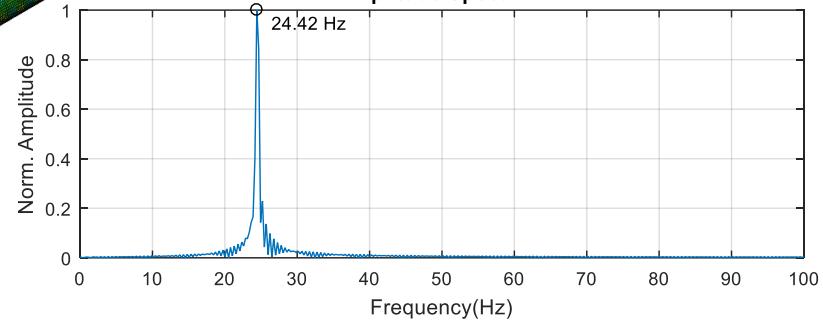
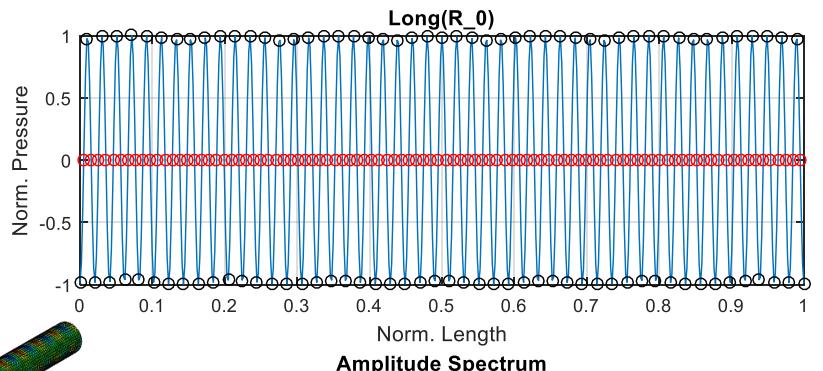
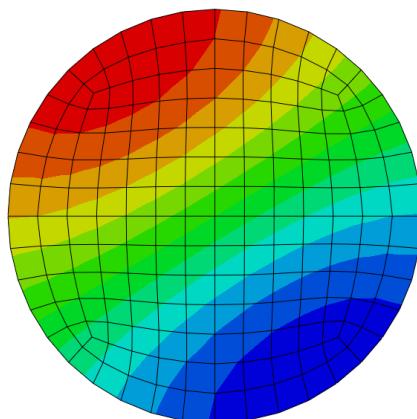
rigid pipe

mode (1,0)

- the last mode shape in 0-40 kHz
 - 39992.8 Hz
 - 49 wavelengths



› wavelength:
 $\lambda = 4.08 \cdot 10^{-2} m$

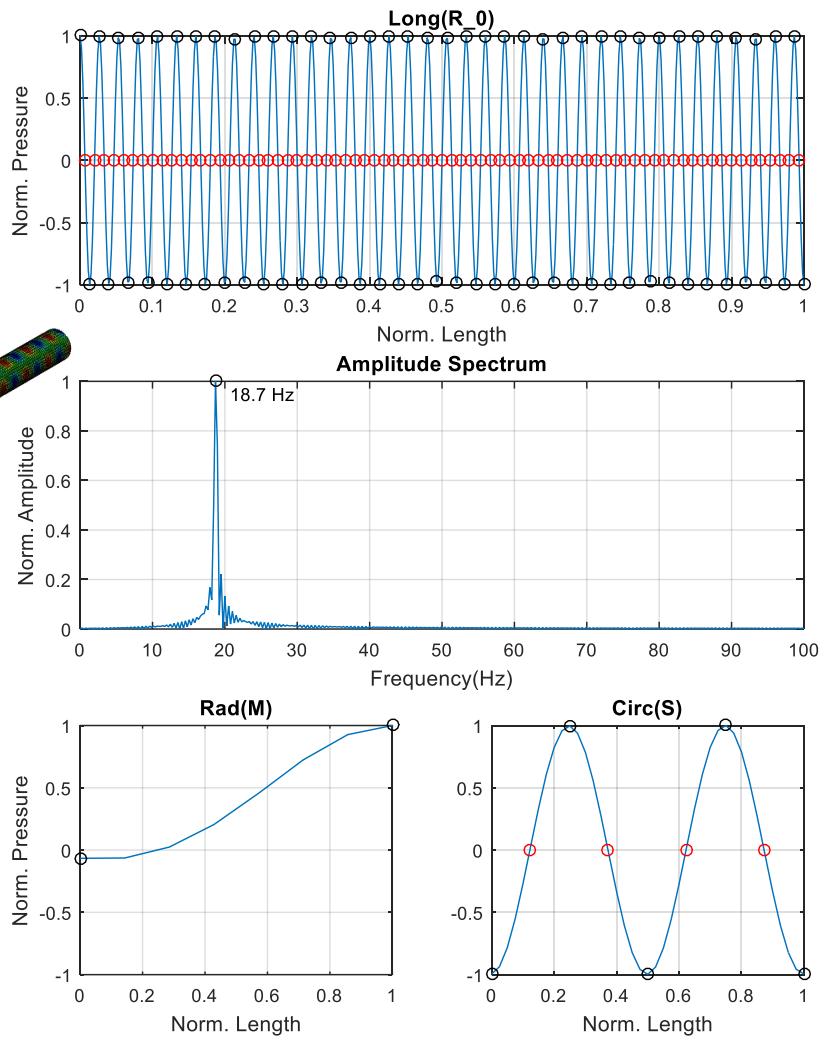
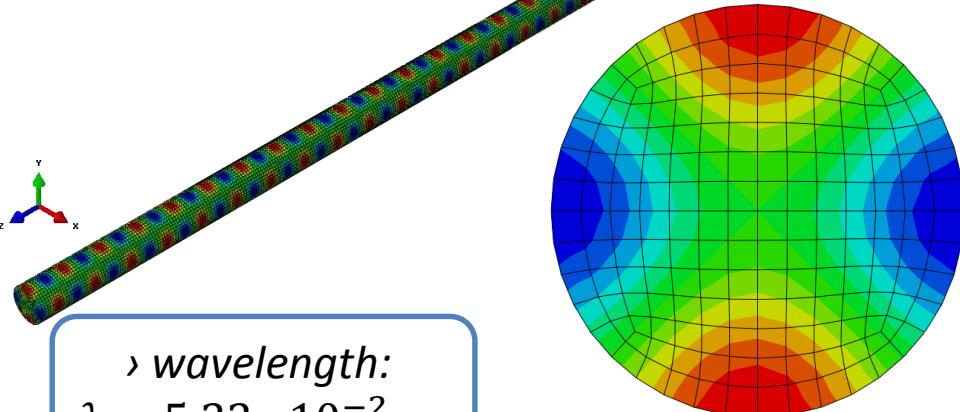
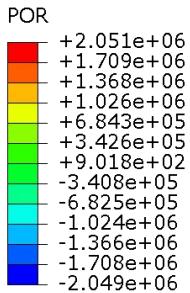


results



rigid pipe mode (2,0)

- the last mode shape in 0-40 kHz
 - 39890.6 Hz
 - 37.5 wavelengths

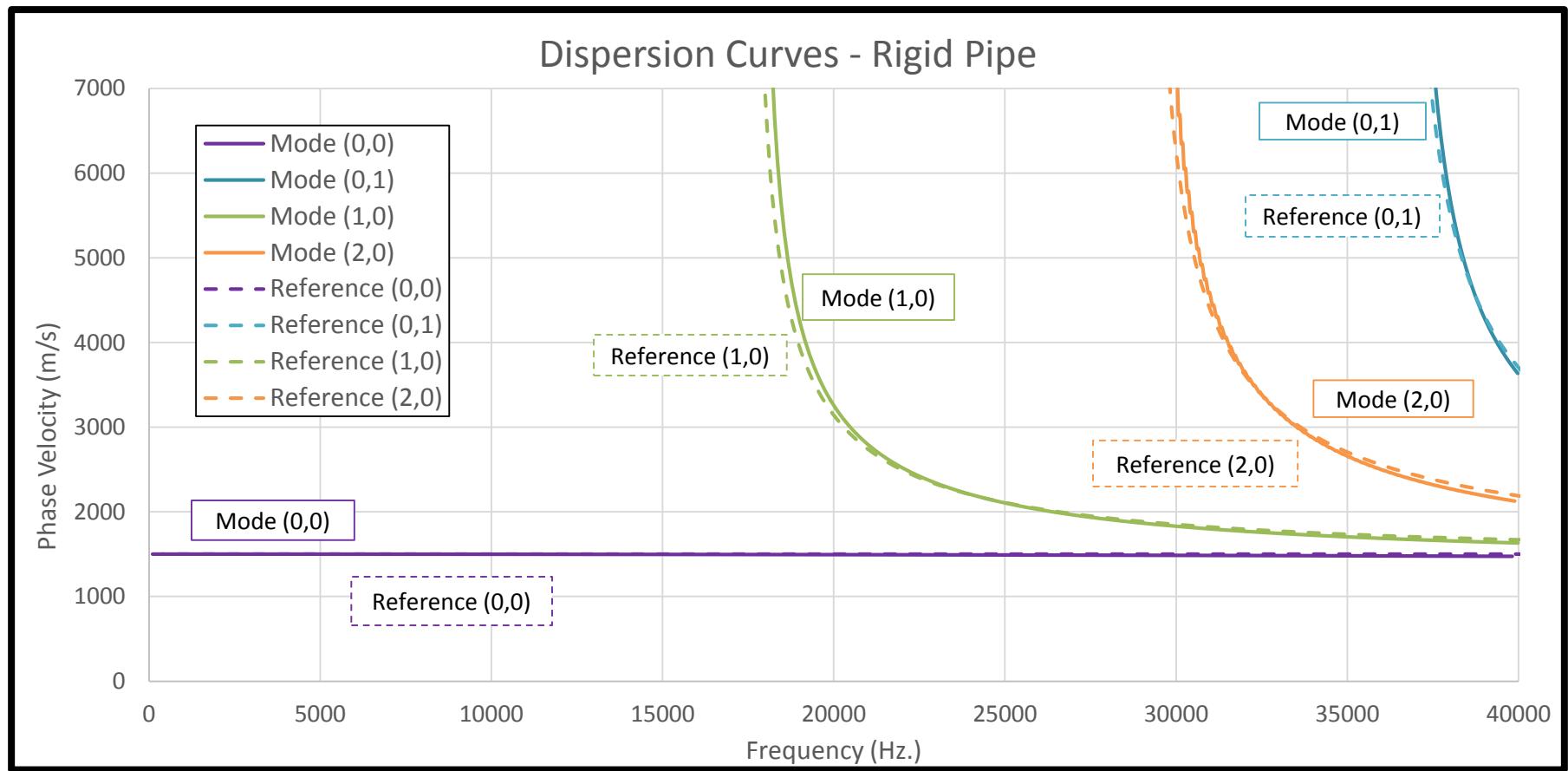


results



rigid pipe

dispersion curves – reference comparison





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FEM model

mode identification

rigid pipe

deformable pipe

pipe-only



results

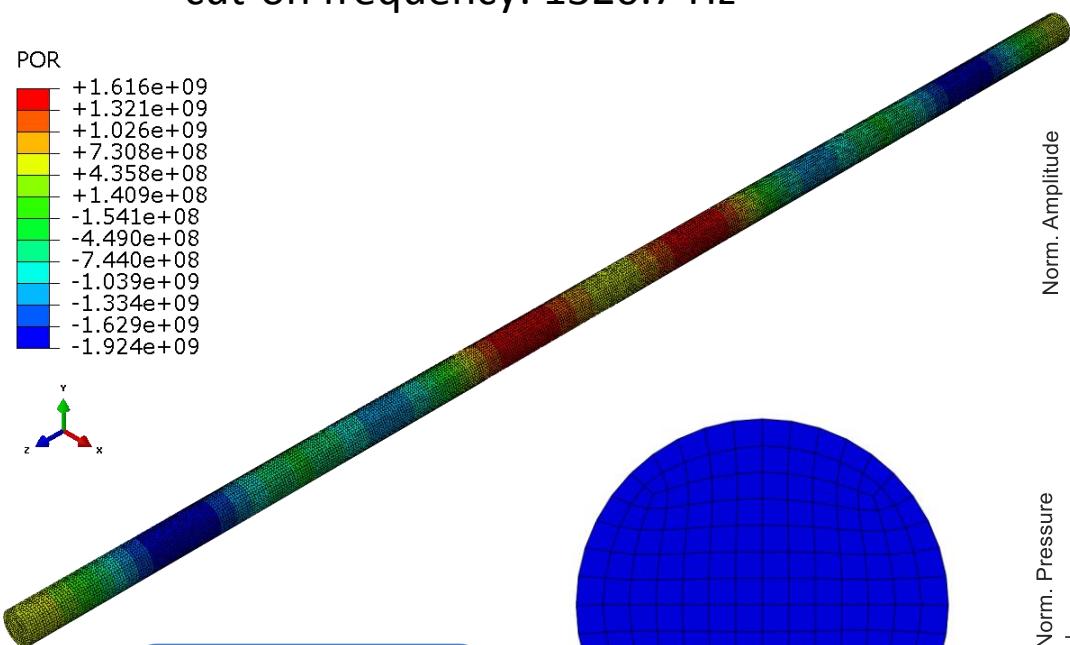
deformable pipe: 0-10 kHz

mode $(0,0)^*$ - water domain

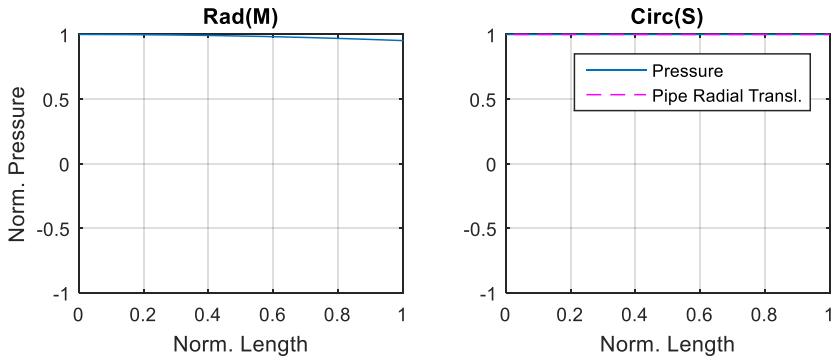
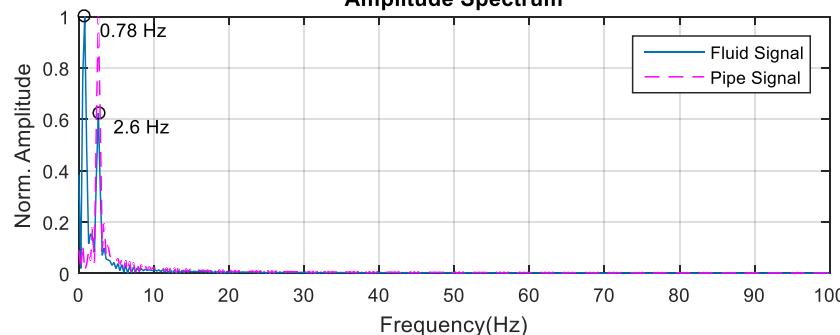
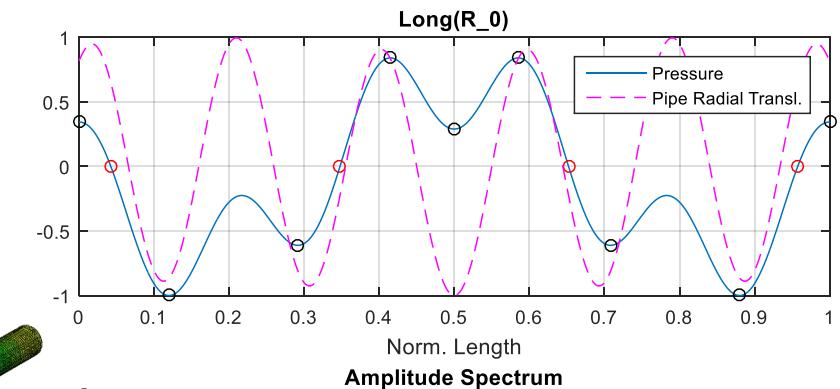
- >1 peaks between consecutive ZCs
- cut-on frequency: 1320.7 Hz

POR

| | |
|--|------------|
| | +1.616e+09 |
| | +1.321e+09 |
| | +1.026e+09 |
| | +7.308e+08 |
| | +4.358e+08 |
| | +1.409e+08 |
| | -1.541e+08 |
| | -4.490e+08 |
| | -7.440e+08 |
| | -1.039e+09 |
| | -1.334e+09 |
| | -1.629e+09 |
| | -1.924e+09 |



› wavelength:
 $\lambda = 1.33 \text{ m}$



results



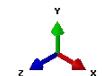
deformable pipe: 0-10 kHz

mode (0,0)* - structure domain

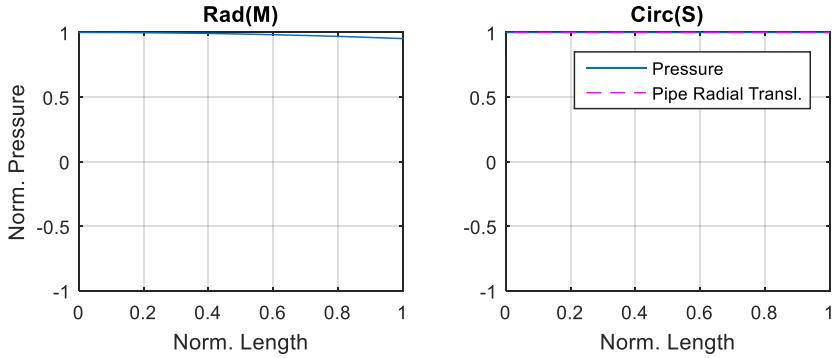
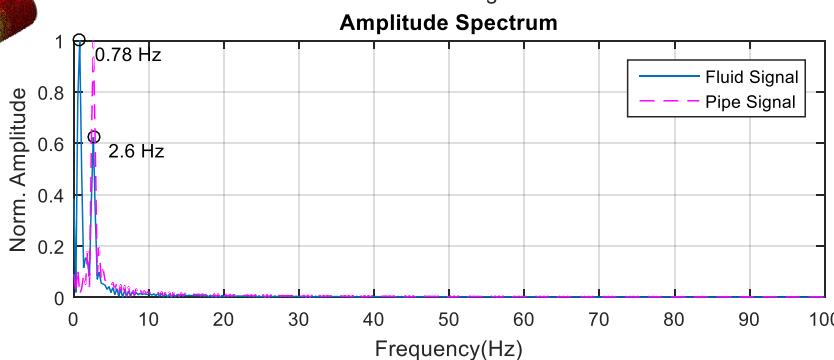
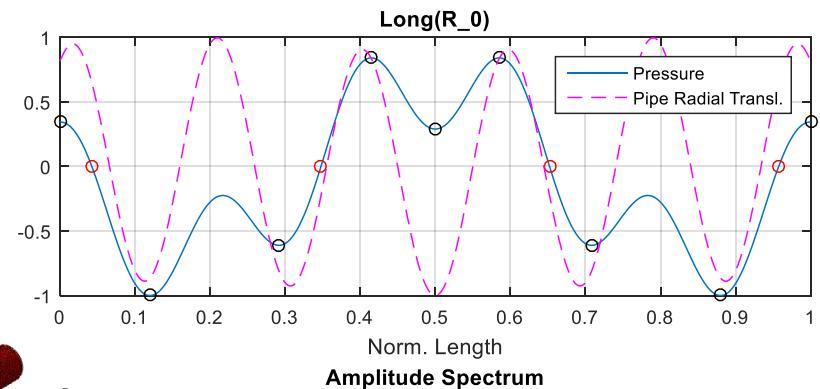
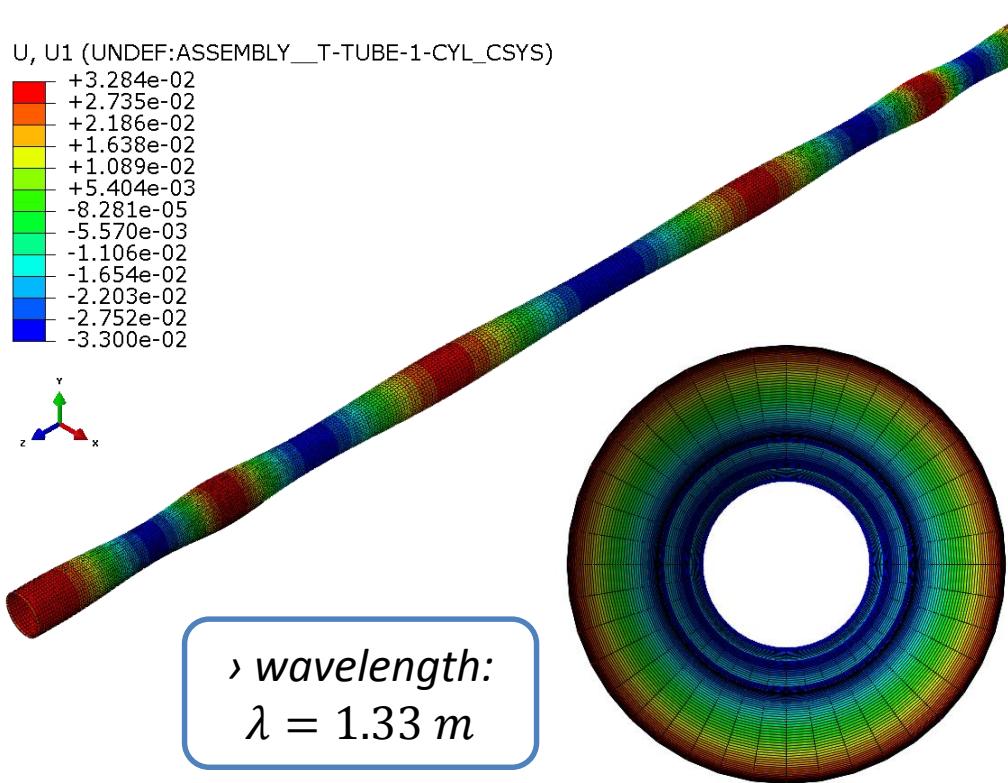
- pressure peaks and translation peaks have opposite signs

U, U1 (UNDEF:ASSEMBLY__T-TUBE-1-CYL_CSYS)

| | |
|---|-----------|
| + | 3.284e-02 |
| + | 2.735e-02 |
| + | 2.186e-02 |
| + | 1.638e-02 |
| + | 1.089e-02 |
| + | 5.404e-03 |
| - | 8.281e-05 |
| - | 5.570e-03 |
| - | 1.106e-02 |
| - | 1.654e-02 |
| - | 2.203e-02 |
| - | 2.752e-02 |
| - | 3.300e-02 |



› wavelength:
 $\lambda = 1.33 \text{ m}$



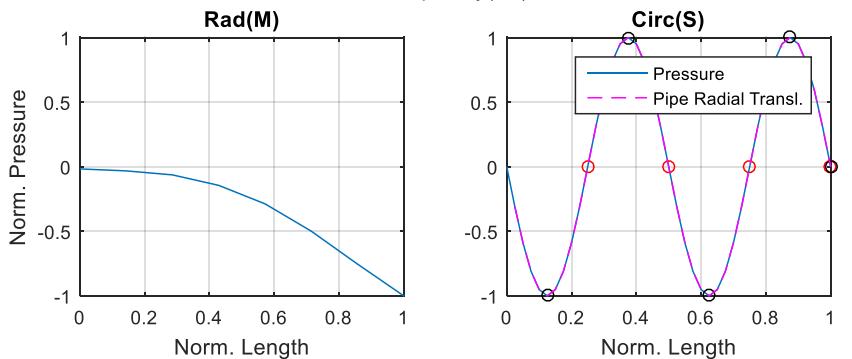
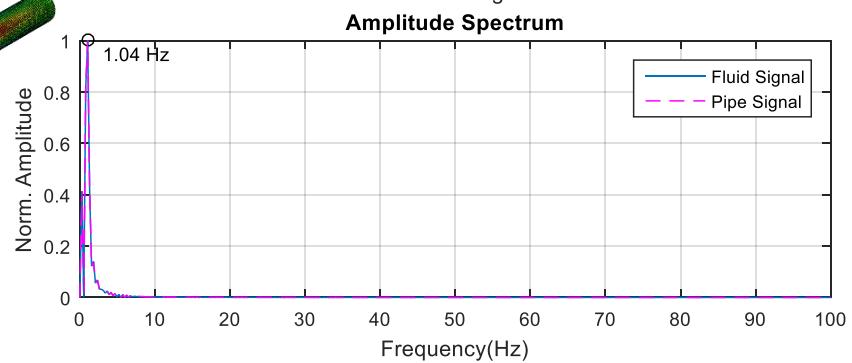
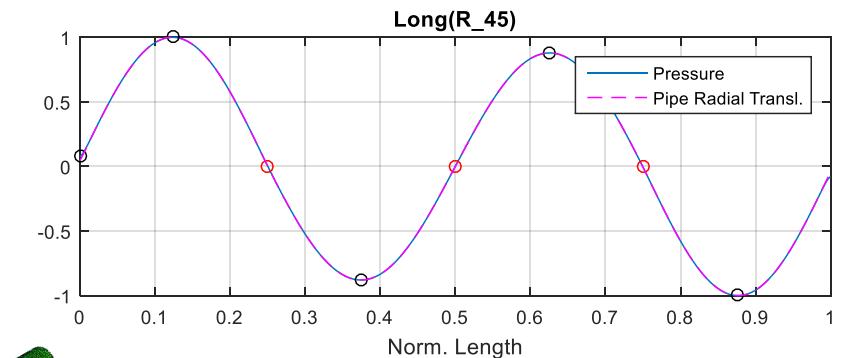
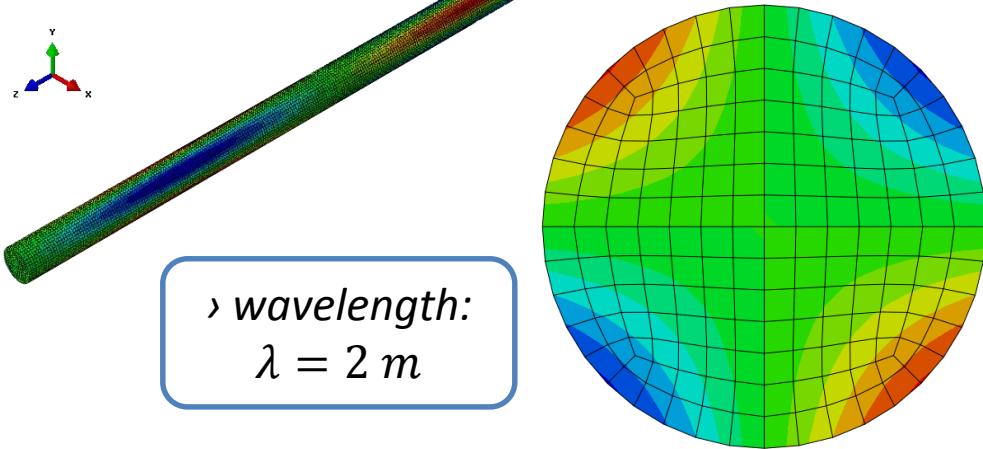
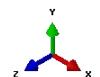
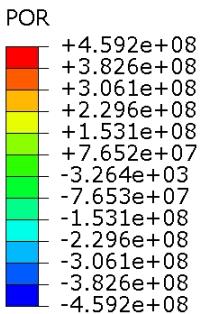


results

deformable pipe: 0-10 kHz

mode (2,0) – water domain

- 2 wavelengths along the circumference
- cut-on frequency: 1042.2 Hz



results



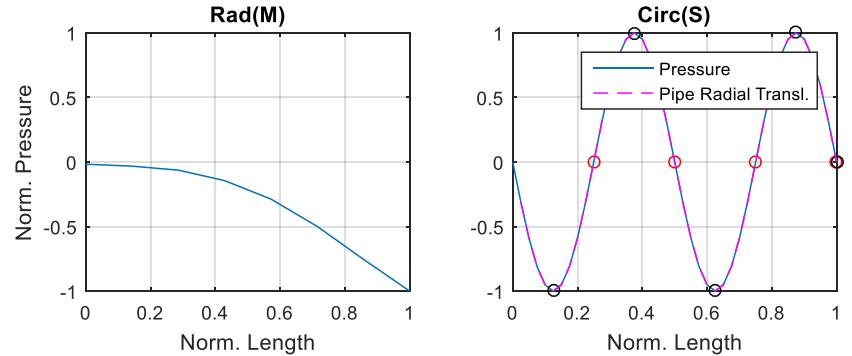
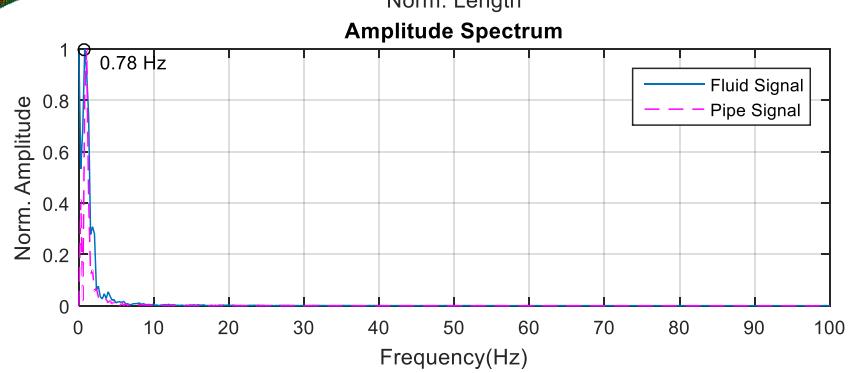
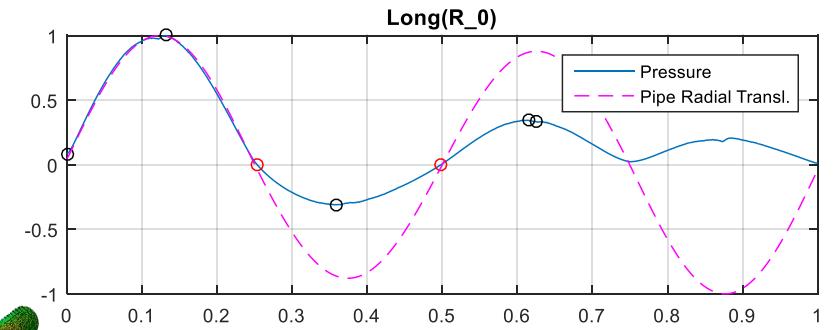
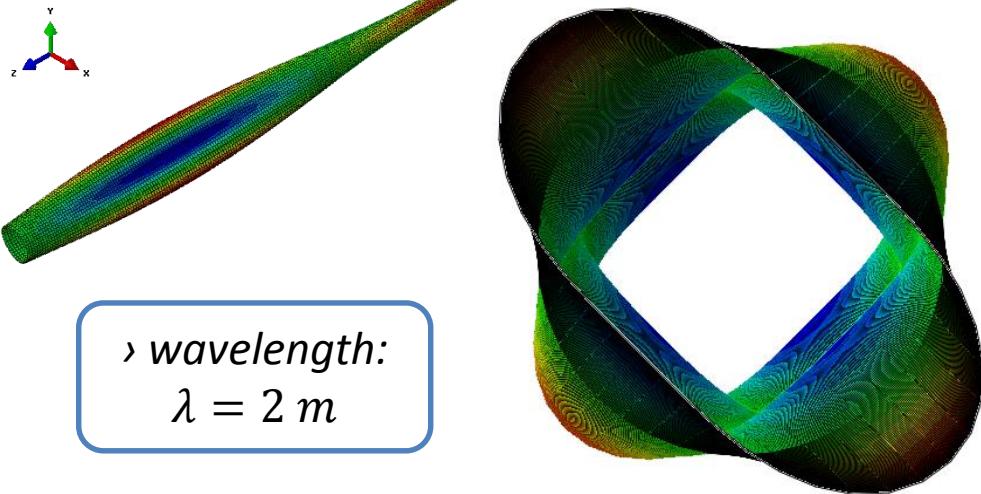
deformable pipe: 0-10 kHz

mode (2,0) – structure domain

- cylindrical R translations shown

U, U_1 (UNDEF:ASSEMBLY__T-TUBE-1-CYL_CSYS)

| |
|------------|
| +8.590e-01 |
| +7.159e-01 |
| +5.727e-01 |
| +4.295e-01 |
| +2.863e-01 |
| +1.432e-01 |
| -1.490e-07 |
| -1.432e-01 |
| -2.863e-01 |
| -4.295e-01 |
| -5.727e-01 |
| -7.159e-01 |
| -8.590e-01 |

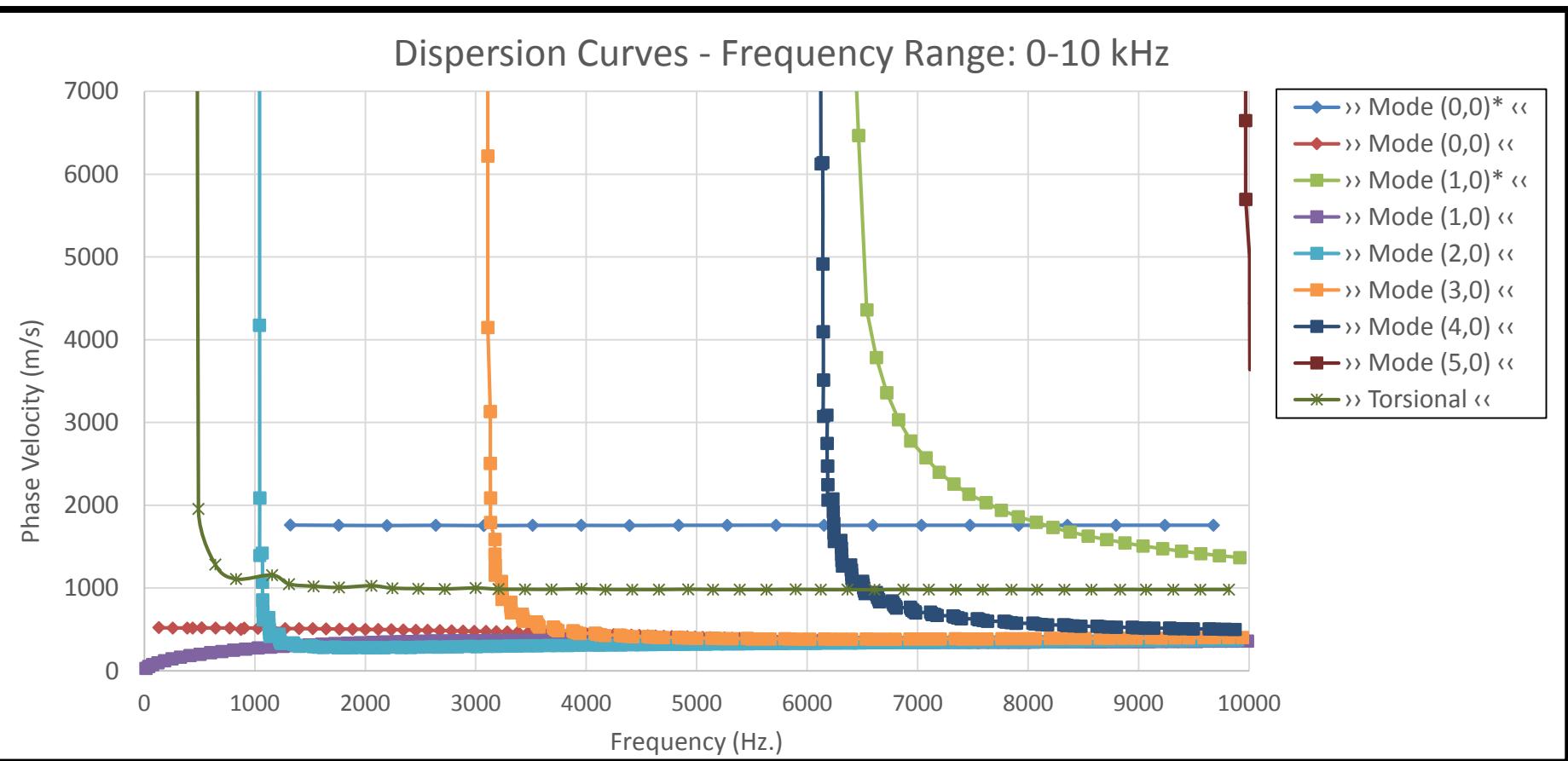




results

deformable pipe: 0-10 kHz

dispersion curves

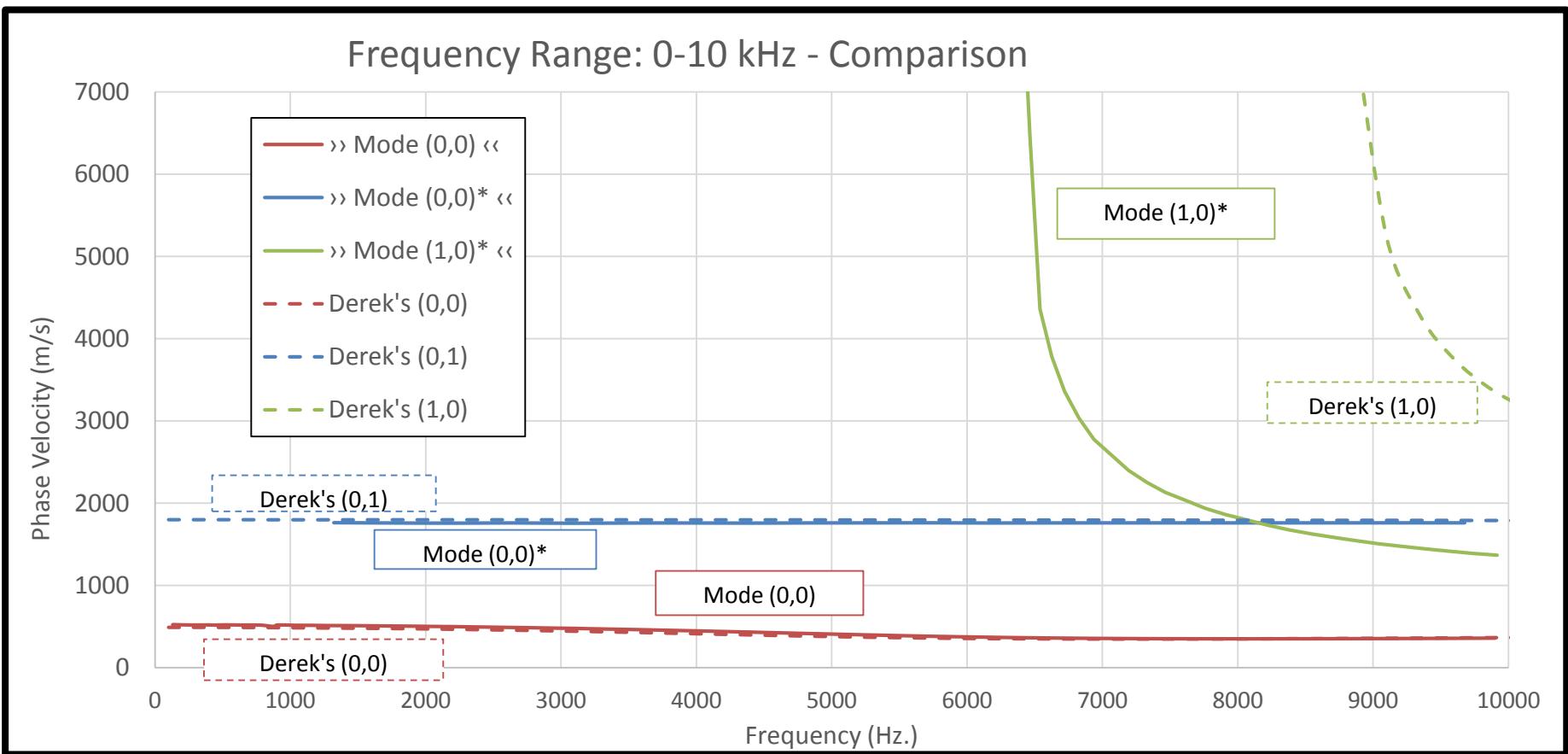




results

deformable pipe: 0-10 kHz

dispersion curves – comparison with Derek's Model

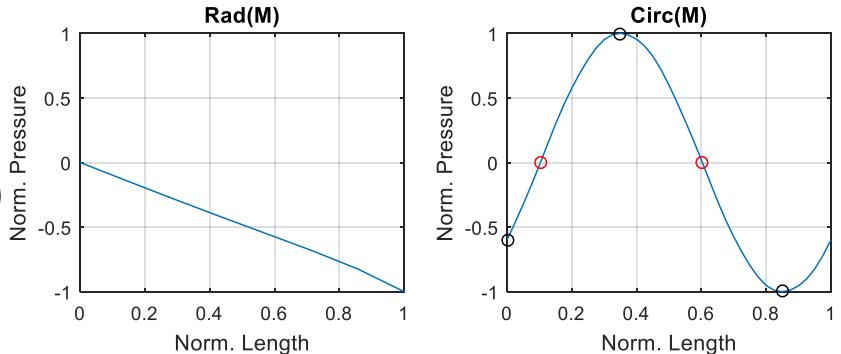
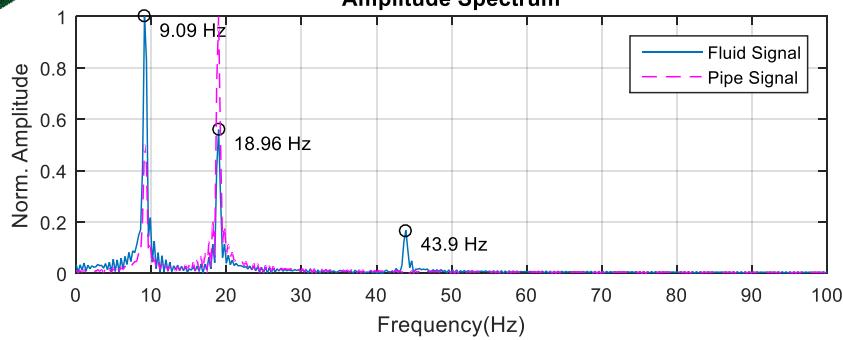
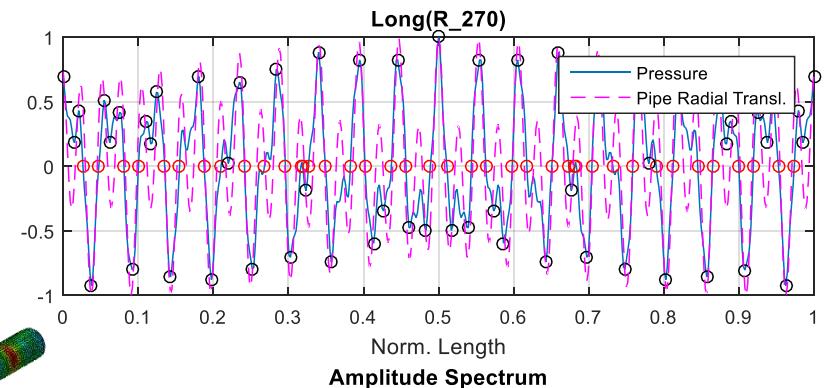
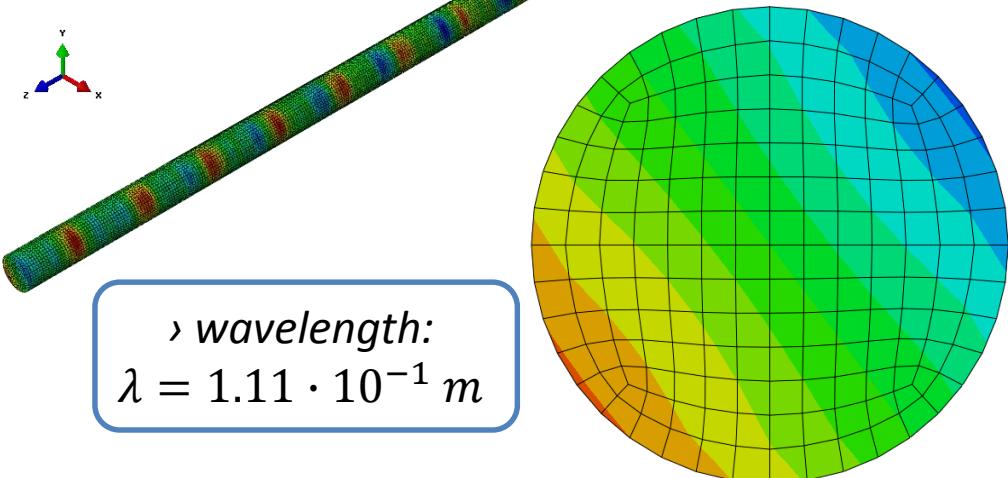
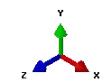
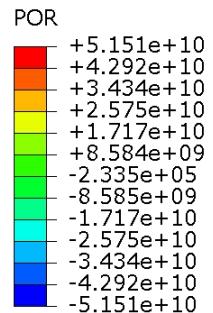


results



deformable pipe: 10-20 kHz mode (1,0)** - water domain

- the last mode shape in 10-20 kHz
 - 19952.2 Hz
 - 18 wavelengths





results

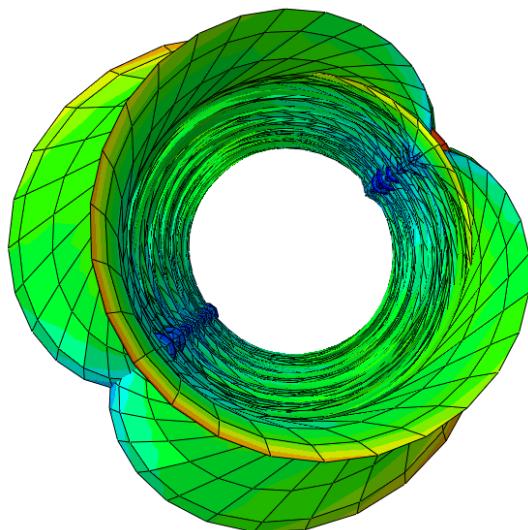
deformable pipe: 10-20 kHz

mode (1,0)** - structure domain

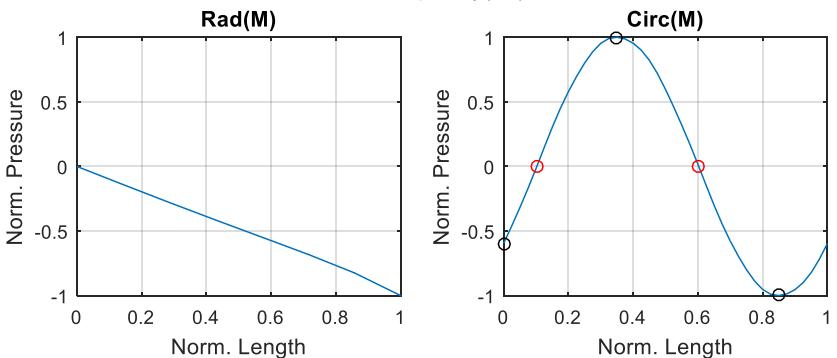
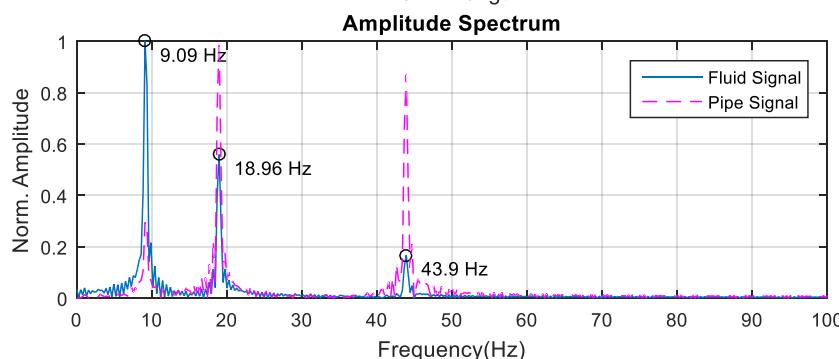
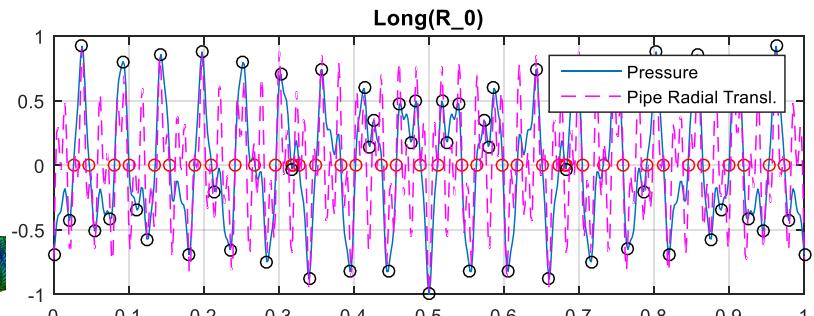
- the last mode shape in 10-20 kHz

U, U1 (UNDEF:ASSEMBLY__T-TUBE-1-CYL_CSYS)

| |
|------------|
| +1.762e-01 |
| +1.468e-01 |
| +1.175e-01 |
| +8.810e-02 |
| +5.875e-02 |
| +2.939e-02 |
| +3.734e-05 |
| -2.932e-02 |
| -5.867e-02 |
| -8.803e-02 |
| -1.174e-01 |
| -1.467e-01 |
| -1.761e-01 |



› wavelength:
 $\lambda = 1.11 \cdot 10^{-1} \text{ m}$

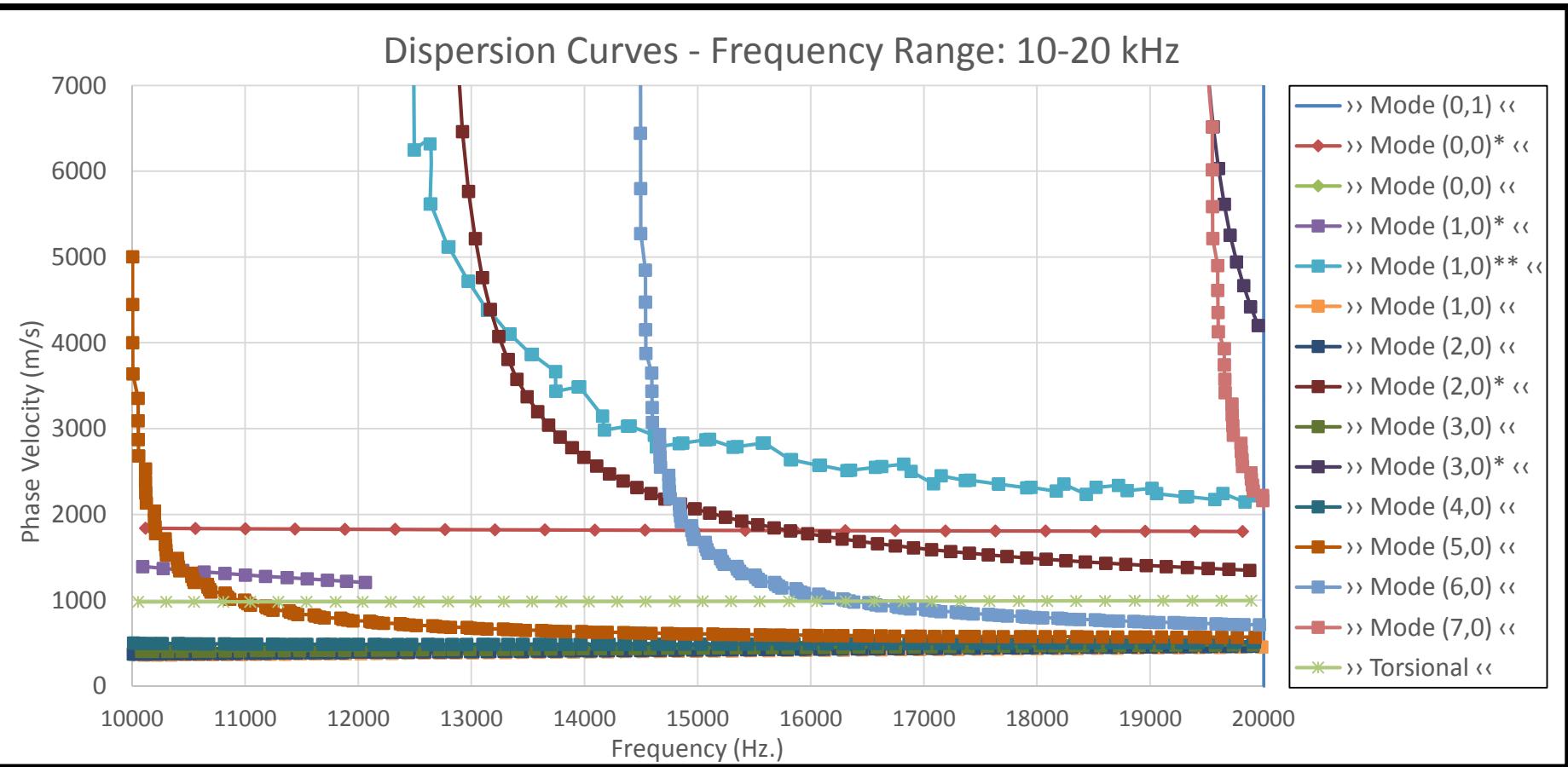




results

deformable pipe: 10-20 kHz

dispersion curves

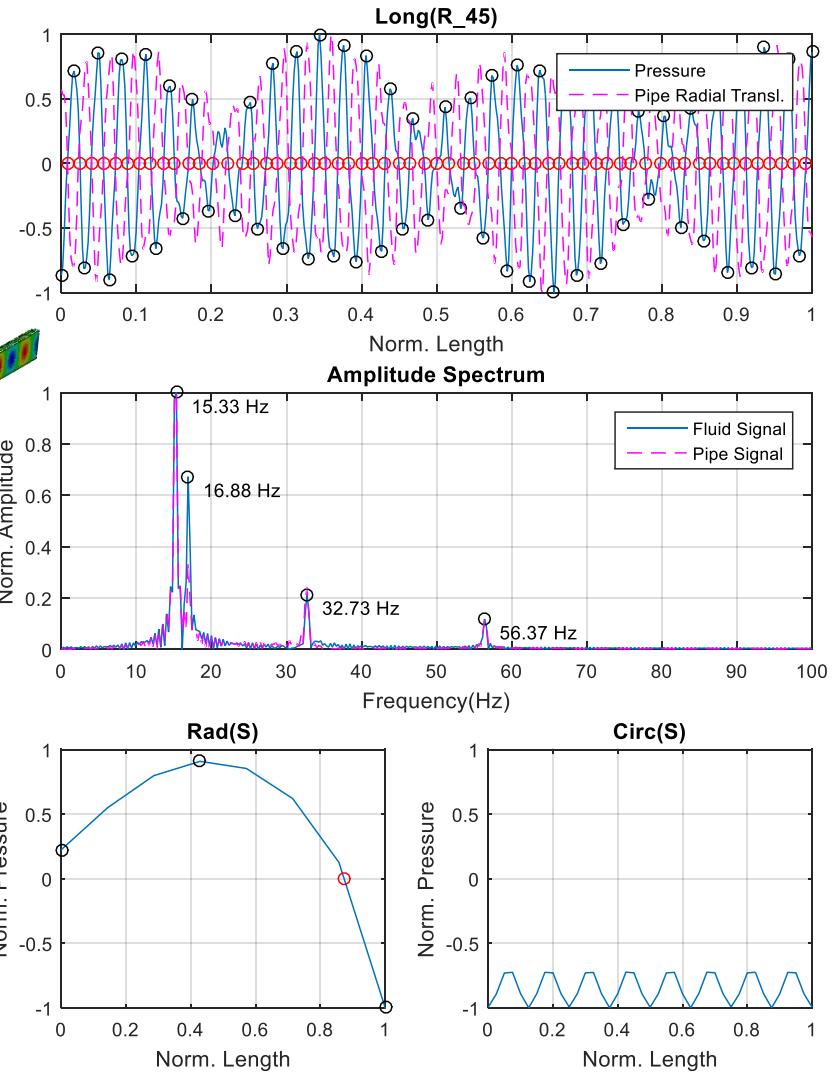
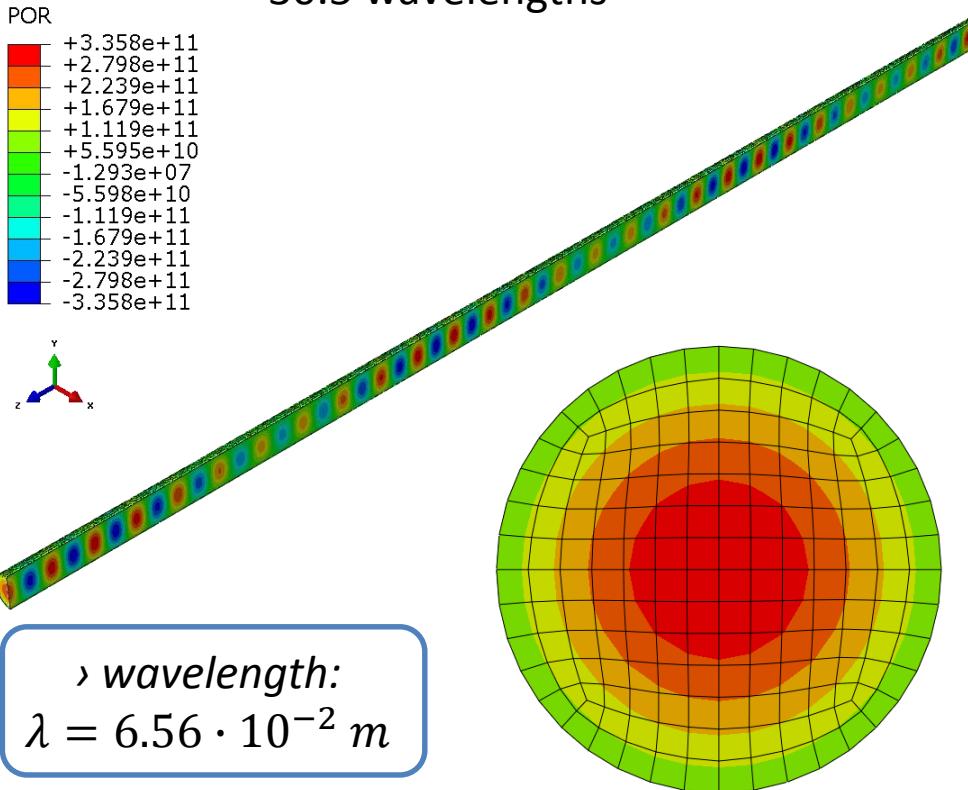


results



deformable pipe: 20-30 kHz mode (0,1) – water domain

- the last mode shape in 20-30 kHz
 - 29893.7 Hz
 - 30.5 wavelengths



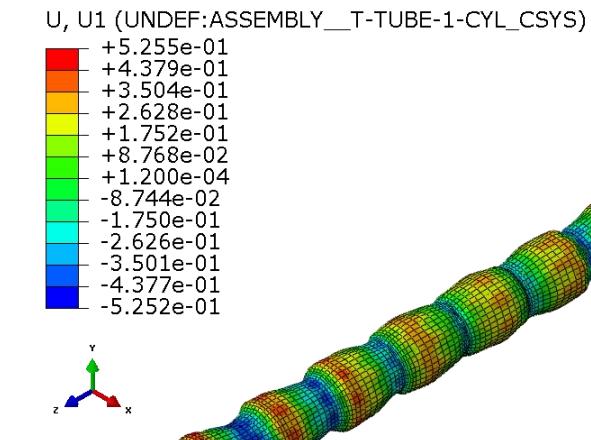


results

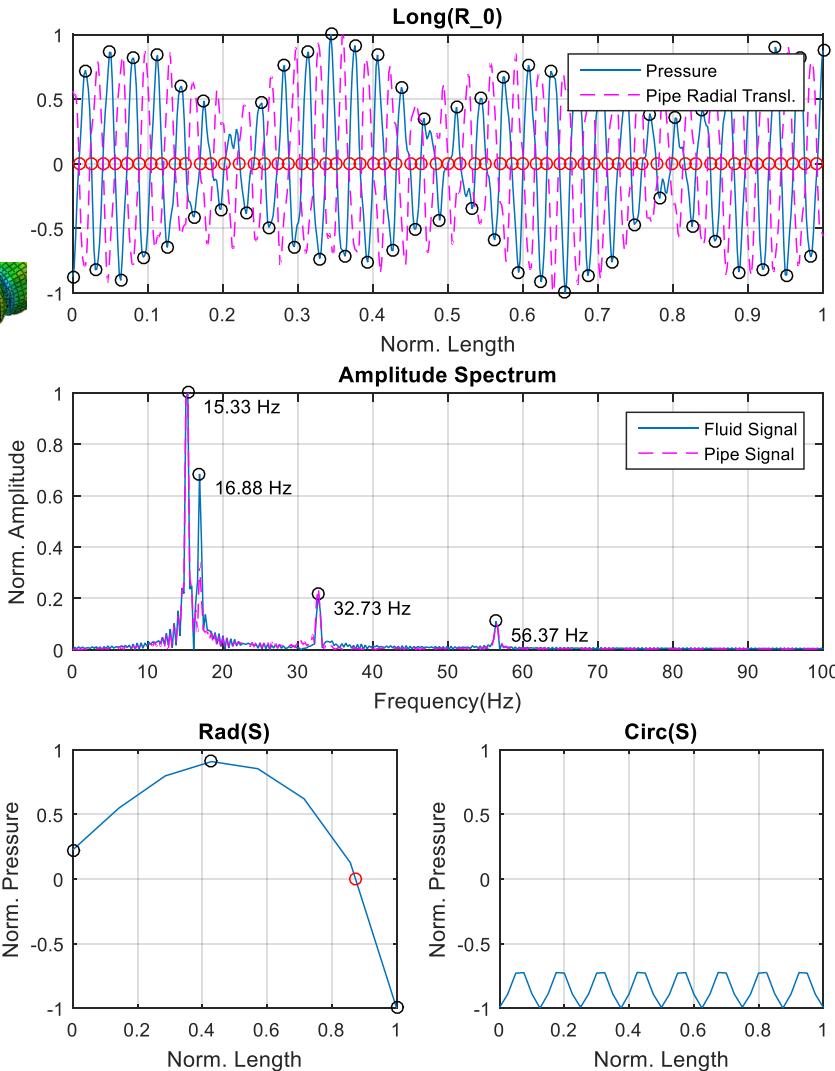
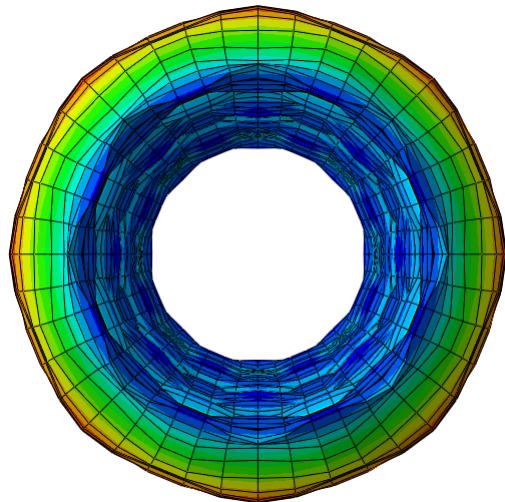
deformable pipe: 20-30 kHz

mode (0,1) – structure domain

- the last mode shape in 20-30 kHz



› wavelength:
 $\lambda = 6.56 \cdot 10^{-2} \text{ m}$

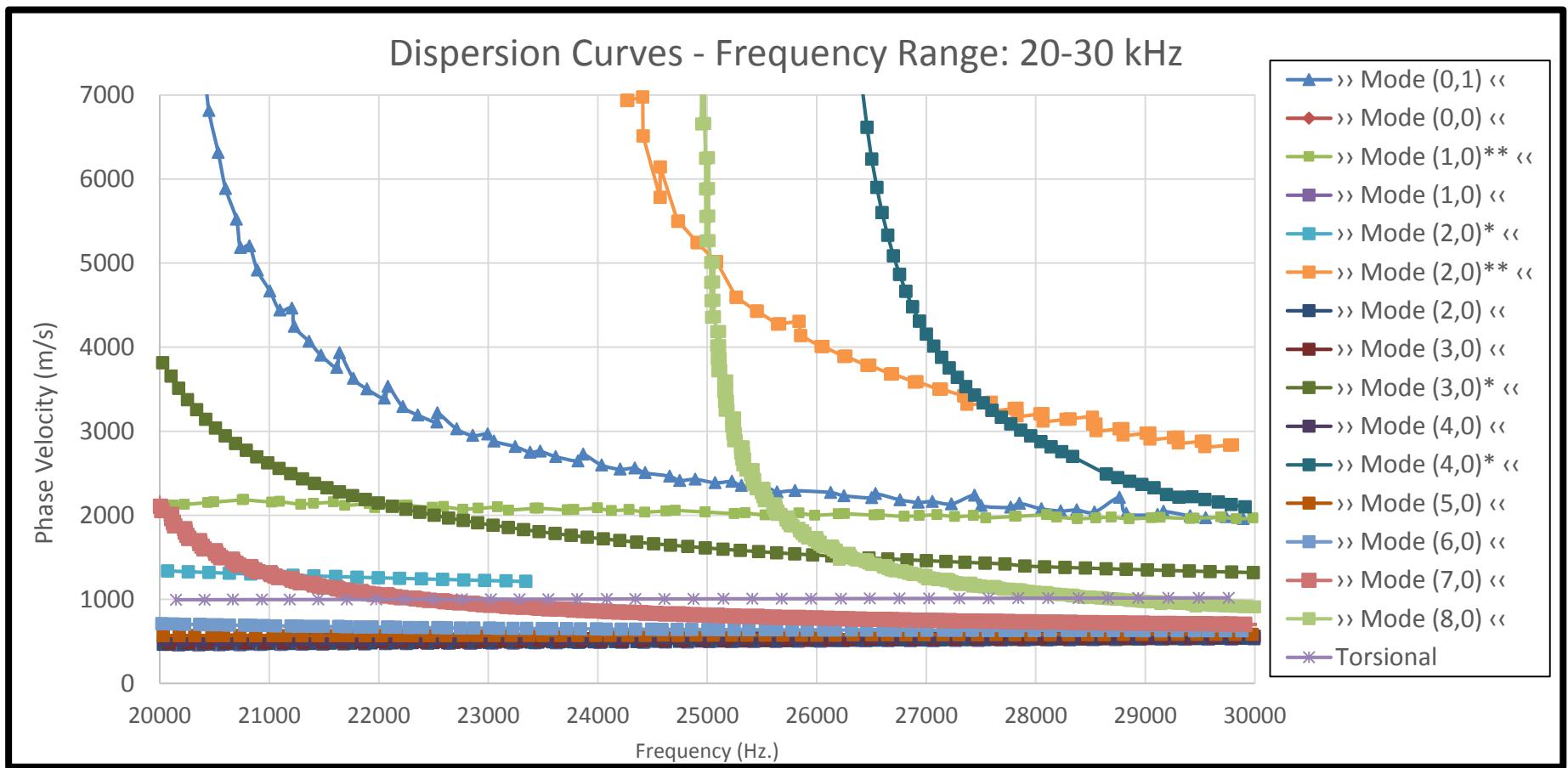




results

deformable pipe: 20-30 kHz

dispersion curves





outline

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FEM model

mode identification

rigid pipe

deformable pipe

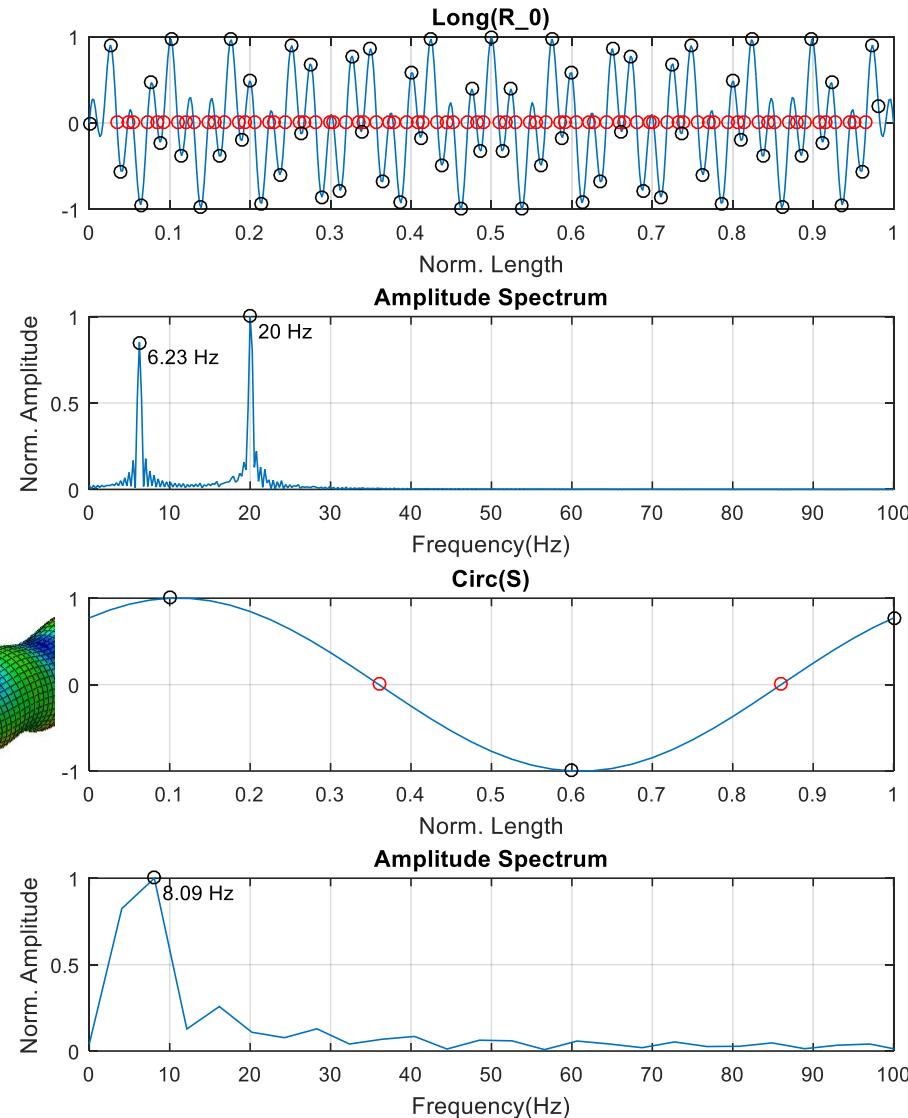
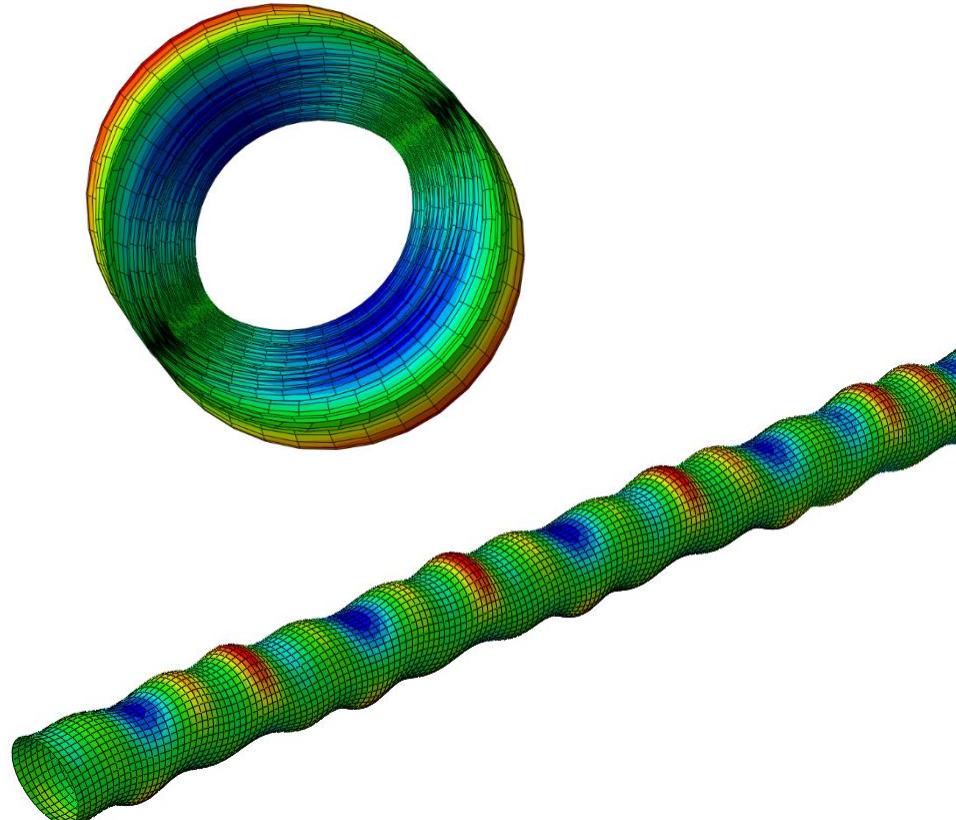
pipe-only

results



pipe-only: 0-10 kHz
mode (1,0)*

- the last mode shape in 0-10 kHz

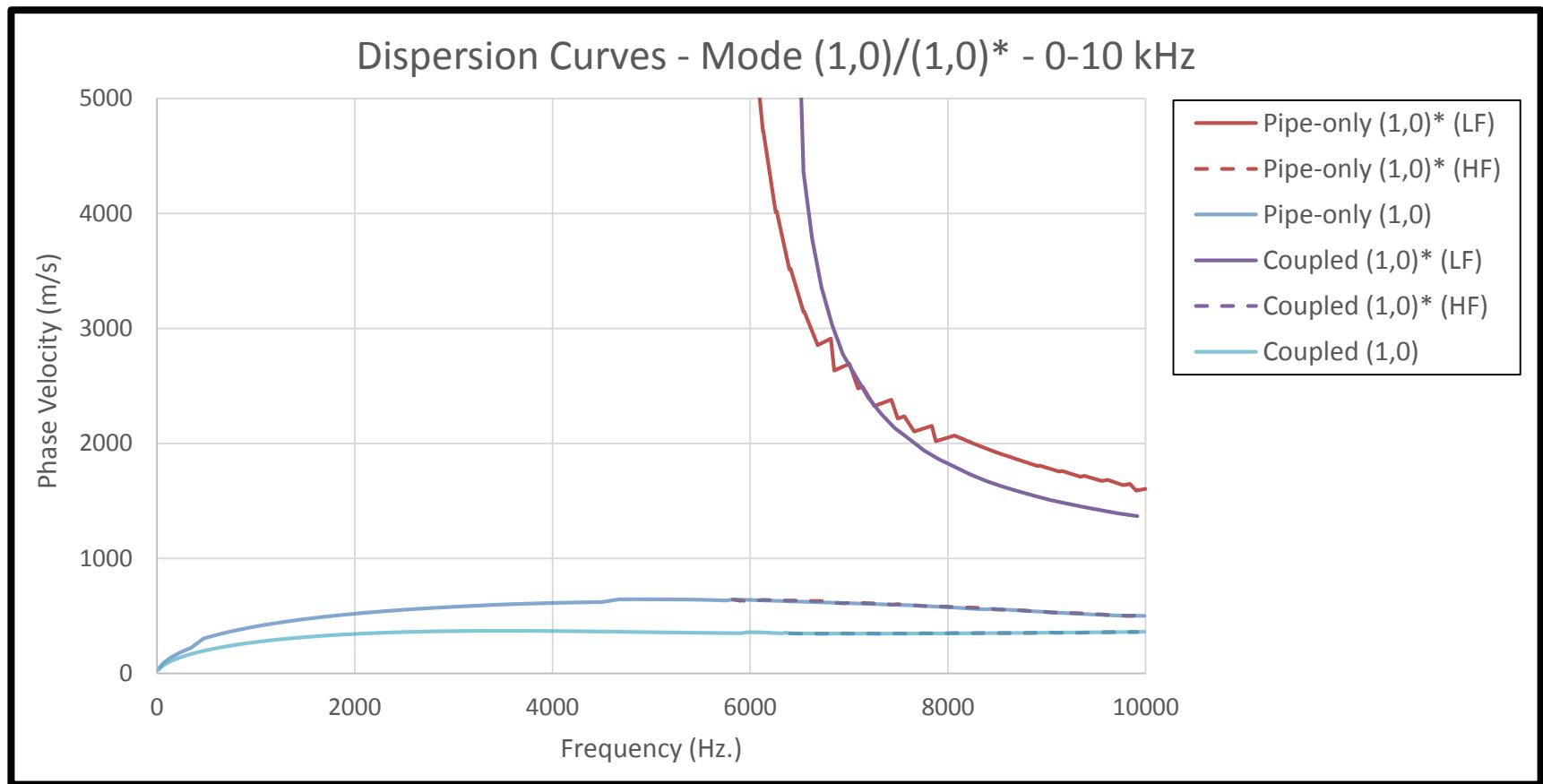


results



pipe-only: 0-10 kHz

comparison with the coupled system

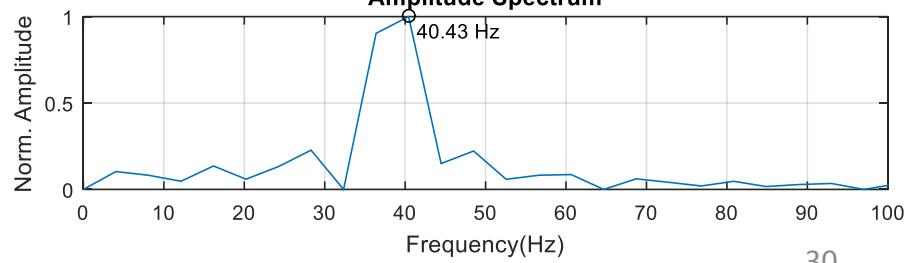
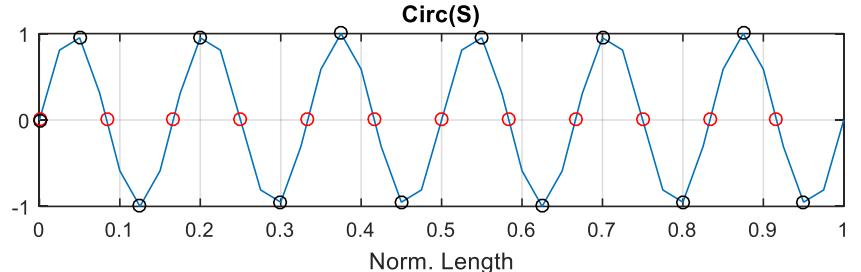
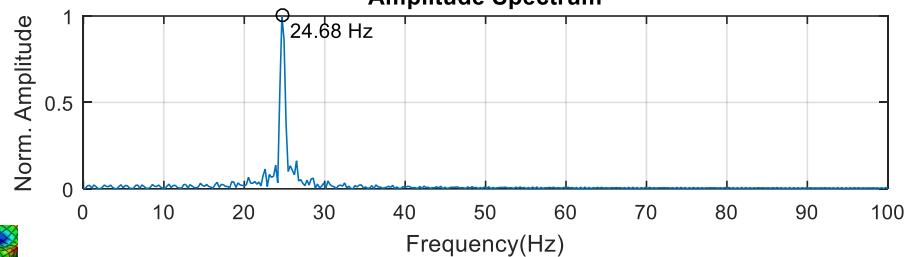
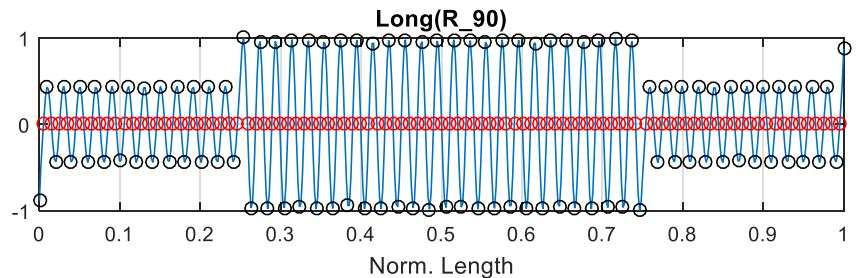
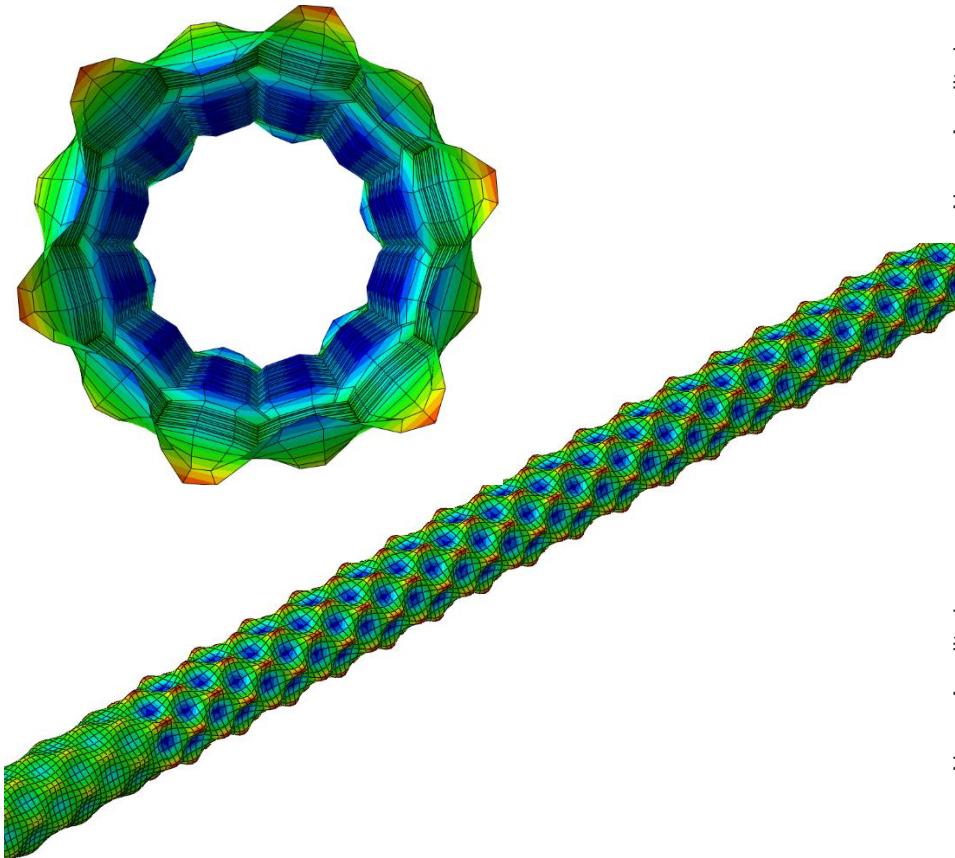


results



pipe-only: 10-20 kHz
mode (6,0)

- the last mode shape in 10-20 kHz



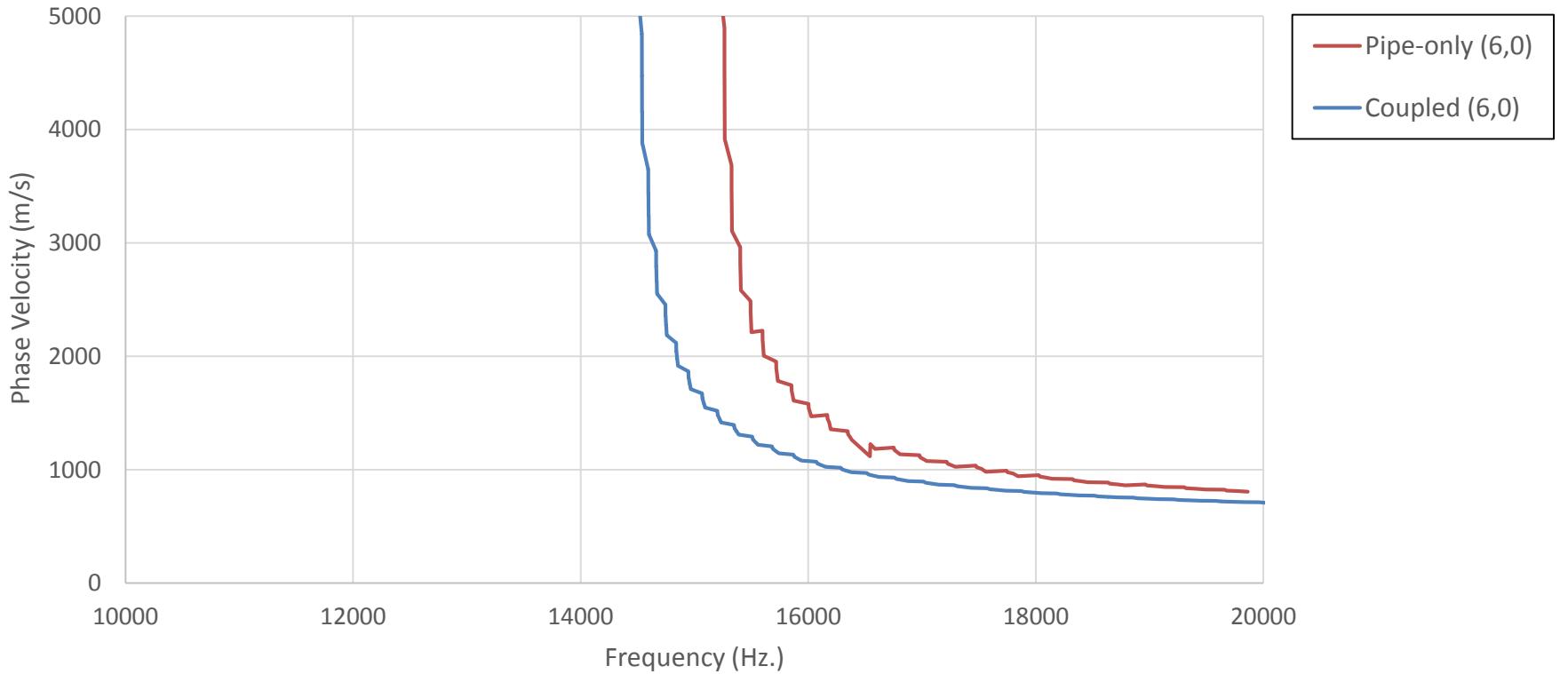
results

pipe-only: 0-10 kHz

comparison with the coupled system



Dispersion Curves - Mode (6,0) - 10-20 kHz





thank you